

## PROPERTIES OF MATTER

### 1.1.INTRODUCTION TO ELASTICITY

*A body can be deformed (i.e. change in shape or size) by applying a suitable force on it. After removing the force, if the body regains its original shape and size, then it is perfectly elastic body.*

*Definition*

*The property on account of which, the body opposes the deforming forces and regains its original shape and size on the removal of the deforming forces is called elasticity.*

*Elastic Body*

*If a body completely regains its original shape and size on the removal of the deforming forces, the body is said to be perfectly elastic. This property of the body is called elasticity.*

*Examples: Rubber and Metals*

### 1.2.DEFORMATION FORCE

The force which changes or tries to change the shape or size of a body without moving it as whole is called deforming force.

### STRESS

When external forces acting on a body produce some deformation of the body, then the forces of reaction is set with in the body to restore the body to its original shape and size.

*The restoring force acting per unit area inside the body is defined as stress.*

The restoring force is equal and opposite to the applied force  $F$ , therefore, stress is also defined as "The deforming force per unit cross sectional area of body"

$$\text{stress} = \frac{\text{Deforming force}}{\text{Area on which the force acts}} = \frac{F}{A}$$

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More appropriate definition of stress is

$$\text{stress} = \frac{\text{Component of Force normal to Area}}{\text{Area on which the force acts}}$$

$$\text{Dimension: } [ML^{-1}T^{-2}] \quad \text{Unit: } N/m^2$$

*Types of Stress*

There are three types of stress (a). Linear or Longitudinal Stress (b). Shearing or Tangential Stress (c). Volume or Bulk Stress

### 1.3.STRAIN

External forces acting on a body produce a change in its shape and size. The fractional change is called strain.

*The change in dimension or shape of a body due to the deforming force results in strain.*

The strain is measured by the ratio of change in dimension to original dimension

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

*Types of Strains*

According to the change in length or area or volume, there are three types of strains namely

- (i) *Linear strain (Change in length per unit length)*
- (ii) *Shearing strain (Change in area per unit area)*
- (iii) *Volume or bulk strain (Change in volume per unit volume)*

*Longitudinal Strain:*

*Definition: It is defined as the ratio between the changes in length to original length, without any change in its shape, after the removal of external forces.*

$$\text{Longitudinal Strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{l}{L}$$

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### Shearing Strain

*Definition: It is defined as the angular deformation produced on the body due to the application of external tangential forces on it.*

$$\text{Shearing Strain} = \frac{\text{Angular deformation}}{\text{Tangential force}}$$

### Volume or bulk strain

*Definition: It is defined as the ratio between changes in volume to the original volume, without any change in its shape.*

$$\text{Volume Strain} = \frac{\text{Change in Volume}}{\text{Original Volume}} = \frac{v}{V}$$

### Elastic limit

*The maximum stress upto which a body exhibits the property of elasticity is called elastic limit or limit of elasticity.*

### 1.4.HOOKE'S LAW

*Robert Hooke an English Physicist in the year 1679 had given a relation between stress and strain. This relation is known as Hooke's law.*

*Statement: It states that within the elastic limit stress in a body is directly proportional to the strain produced on it.*

*i.e., Stress  $\propto$  strain.*

$$\frac{\text{stress}}{\text{strain}} = \text{constant}$$

$$E (\text{Constant}) = \frac{\text{stress}}{\text{strain}}$$

In other words the ratio between stress and strain is a constant. *This constant of proportionality is known as coefficient of elasticity or modulus of elasticity.* It has the units of stress i.e.  $N/m^2$  and dimensions are  $[ML^{-1}T^{-2}]$ .

## 1.5.STRESS -STRAIN DIAGRAM

Consider a wire which is rigidly fixed at one end. It is loaded at the other end. The strains produced for the different loads are noted until the wire breaks down. This graph is known as stress-Strain diagram or graph.

The following useful information regarding the behavior of solid materials is studied from this diagram.

### 1. Hooke's Law

The portion OA of the curve is a straight line. In this region, stress is directly proportional to strain. This means that up to OA, the material obeys Hooke's law. The wire is perfectly elastic, the point A is called the limit of proportionality.

### 2. Elastic limit

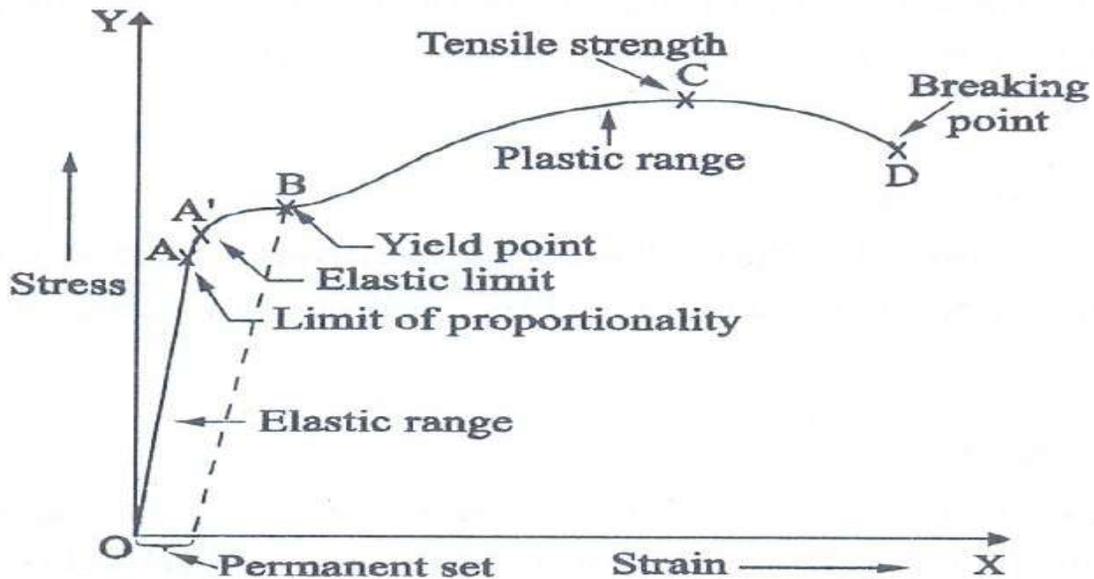
The stress is further increased till a point A'. The point A', lying near to A denotes the elastic limit. Upto the point A' the wire regains its original length if the stress is removed. If the wire is loaded beyond the elastic limit, then it will not return its original length.

### 3. Yield point

On further increasing the stress beyond the elastic limit, the curve bends and a point B is reached.

In this region A'B, a slight increase in stress produces a larger strain in the material. The point B is called yield point. The value of stress at the point is called yield strength of the material.

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#### 4. Permanent set

In the region A' B, if stress is removed, the wire will never return to its original length. The wire is taken a permanent set.

#### 5. Plastic range

Beyond B the strain in the wire increases rapidly without any increase in the load. This is known as plastic range.

#### 6. Ultimate tensile strength

If the wire is further loaded, appoint C is reached after which the wire begins to neck down. Hence its cross sectional area is no longer remains uniform.

At the point C, the wire begins to thin down at the same point and it finally breaks. At the point C, the stress developed is maximum and it is called Ultimate tensile strength

#### 7. Breaking point.

The point 'B' is known as the breaking point where the wire breaks down completely. The stress at the point D is called breaking point.

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## 1.6. TYPES OF MODULI OF ELASTICITY

There are three types of moduli of elasticity corresponding to three types of strains. They are

- Young's Modulus of elasticity corresponding to linear strain.
- Rigidity Modulus or shear modulus of elasticity corresponding to shear strain
- Bulk Modulus or volume modulus of elasticity corresponding to volume strain

### 1.6.1. Young's Modulus of Elasticity (Y)

Within the elastic limit, the ratio of linear stress to linear strain is called Young's Modulus of Elasticity.

$$\text{Young's Modulus of Elasticity (Y)} = \frac{\text{Linear stress}}{\text{Linear strain}}$$

The linear force  $F$  is applied normally to a cross sectional area " $a$ " of a wire

$$\text{Linear stress} = \frac{\text{Linear Force}}{\text{Cross sectional area}} = \frac{F}{a}$$

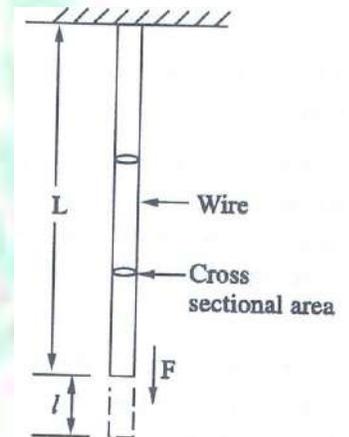
If  $L$  is the original length and  $l$  is the change in length due to the applied force, then

$$\text{Linear strain} = \frac{\text{change in length}}{\text{Original length}} = \frac{l}{L}$$

$$\text{Young's Modulus of Elasticity (Y)} = \frac{\text{Linear stress}}{\text{Linear strain}}$$

$$Y = \frac{\frac{F}{a}}{\frac{l}{L}} = \frac{FL}{al}$$

$$\text{Young's Modulus of Elasticity (Y)} = \frac{FL}{al} \quad \text{Unit : } N m^{-2}$$



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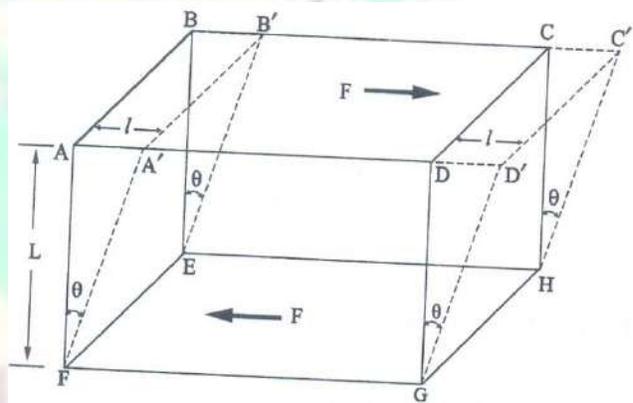
### 1.6.2. Rigidity Modulus or shear modulus of elasticity

Within the elastic limit, the ratio of tangential stress to shearing strain is called Rigidity Modulus of Elasticity. It is denoted by letter  $n$

$$\text{Rigidity Modulus } (n) = \frac{\text{Tangential Stress}}{\text{Shearing Strain}}$$

Consider a rectangular block fixed at its lower face EFGH. A force "F" is applied tangentially on the upper face ABCD as shown in figure.

A force of reaction of the same magnitude F acts on the lower face EFGH in the opposite direction. The two equal and opposite forces form a couple. Due to this couple, the body gets deformed and its shape changes shown on figure.



$$\begin{aligned} \text{Shearing Stress} &= \frac{\text{Tangential force}}{\text{Area of the Face ABCD}} \\ &= \frac{F}{A} \end{aligned}$$

All the four vertical sides are twisted through an angle  $\theta$ . This angle  $\theta$  is known as the shearing strain or angle of shear

From the figure  $\tan \theta = \frac{AA'}{AF} = \frac{l}{L}$

$$\text{Shearing Strain } \theta = \frac{l}{L}$$

$$\text{Rigidity Modulus } (n) = \frac{\text{Tangential Stress}}{\text{Shearing Strain}}$$

$$\text{Rigidity Modulus } (n) = \frac{\frac{F}{A}}{\frac{l}{L}} = \frac{FL}{Al} \quad \text{Unit : } N m^{-2}$$

### 1.6.3. Bulk Modulus of elasticity (K)

Within the elastic limit, the ratio of Volume stress to Volume strain is called Bulk Modulus of Elasticity. It is denoted by letter K.

$$\text{Bulk Modulus } (K) = \frac{\text{Volume Stress}}{\text{Volume Strain}}$$

When a deforming force F acts normally on all the faces of the solid body, the body undergoes a change in its volume but not in the shape. This body suffers decrease in volume under the action of the forces.

Volume of the Body = V

Surface area of each face subjected to the force =

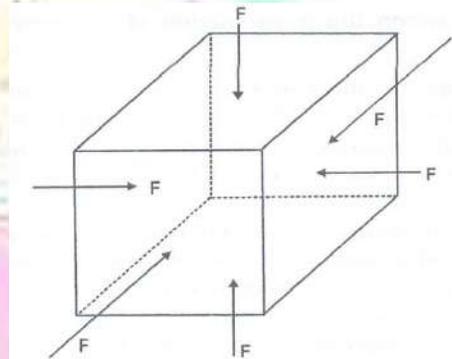
A

Change in Volume = v

$$\text{Volume Stress} = \frac{\text{Normal force}}{\text{Area}} = \frac{F}{A} = P$$

$$\text{Volume Strain} = \frac{\text{Change in Volume}}{\text{Original volume}} = \frac{v}{V}$$

$$\text{Bulk Modulus } (K) = \frac{\text{Volume Stress}}{\text{Volume Strain}} = \frac{P}{\frac{v}{V}} = \frac{PV}{v} \quad \text{Unit : } N m^{-2}$$



### 1.7. RELATIONSHIP BETWEEN THREE MODULI OF

ELASTICITY (Qualitative) (Modulus of Elasticity)

There are many relations connecting to the lateral strain ( $\beta$ ), longitudinal strain ( $\alpha$ ), Poisson's ratio ( $\sigma$ ) and the three elastic moduli. Some of the relations are given below.

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- (i) Relation between  $\alpha$  and young's modulus is  $\alpha = \frac{1}{Y}$
- (ii) Relation between  $\alpha$  and  $\beta$  with the Bulk modulus  $\alpha - 2\beta = \frac{1}{3K}$
- (iii) Relation between  $\alpha$  and  $\beta$  with the Rigidity modulus  $(\alpha + \beta) = \frac{1}{2n}$
- (iv) Relation between  $Y$ ,  $n$  and  $K$  is  $Y = \frac{9Kn}{3K+n}$
- (v) Relation between,  $K$  and  $\sigma$  is  $\alpha = \frac{3K-2n}{6K+2n}$
- (vi) Relation between  $Y$ ,  $n$  and  $\sigma$  is  $\sigma = \frac{Y}{2n} - 1$

### 1.8. POISSON'S RATIO

It is a commonly observed fact that when we stretch a string or a wire, it becomes longer but thinner. A longitudinal strain produced in the wire is accompanied by a transverse or lateral strain of an opposite kind in a direction at right angles to the direction of the applied force.

Within elastic limit, the lateral strain is proportional to the longitudinal strain. The constant of proportionality  $\sigma$  is called as the Poisson's Ratio  $\sigma = l/a$

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

Two more constants are defined as,

$$\alpha = \frac{\text{longitudinal strain}}{\text{longitudinal stress}}$$

$$\beta = \frac{\text{lateral strain}}{\text{longitudinal stress}}$$

$$\text{Hence } \alpha = \frac{1}{Y} \text{ and } \sigma = \frac{\beta}{\alpha}$$

### 1.9. FACTORS AFFECTING ELASTIC MODULES AND TENSILE STRENGTH

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Some materials lose their elastic property even within the elastic limit, due to elastic fatigue. For Example substances like Quartz, Phosphor, Bronze etc. may be employed in manufacturing of galvanometers, electrometers etc., after knowing their properties.

Apart from elastic fatigue, some materials will have change in their elastic property because of the following factors.

*The following factors will change the elastic property of the materials.*

- (i) Effect of stress
- (ii) Effect of annealing
- (iii) Effect of Change in temperature
- (iv) Effect of impurities
- (v) Effect of nature of crystals

Effect of stress:

We know that when a material is subjected to large number of cycles of stresses, it loses its elastic property even within the elastic limit. Therefore the working stress on the materials should be kept lower than the ultimate tensile strengthening and safety factor should be maintained.

Effect of annealing:

Annealing is a process by which the material is heated to a very high temperature and then it is slowly cooled. This process increases the softness and ductility of the material. If annealing is made to a material it results in the formation of large grains, which reduces the elastic property of the material.

Effect of Change in temperature:

The elastic property of the material changes with the temperature. The elasticity increases with the decrease in temperature and vice-versa.

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Example: The elastic property of lead increases when the temperature is decreased. The carbon filament becomes plastic at higher temperature.

Effect of impurities:

The addition of impurities produces variation in the elastic property of the materials. The increase and decrease of elasticity depends on the type of impurity added to it.

Example: When the potassium is added to gold, the elastic property of gold increases. When carbon is added to molten iron the elastic property of iron decreases.

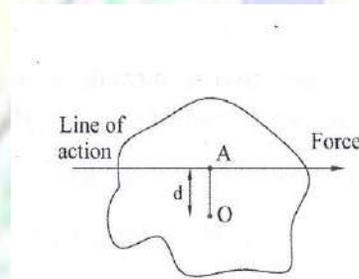
Due to nature of crystals:

The elasticity also depends on the types of the crystals, whether it is a single crystal or poly crystals. For a single crystal the elasticity is more and for a poly crystal the elasticity is less.

## 1.10. MOMENT, COUPLE AND TORQUE

(i). Moment of a force:

*The moment of a force about a point is defined as the product of the magnitude of the force and perpendicular distance from the point to the line of action of force.*

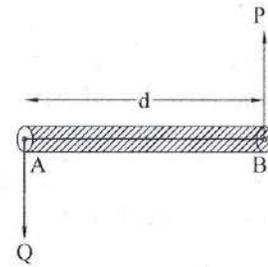


Explanation: Let "F" be the force acting on a body, at A as shown in figure. Then, the moment of force F about "O" is  $M = F \times d$ . Where d is the perpendicular distance from the point to the line of action of force F.

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(ii) Couple:

A couple constitutes a pair of two equal and opposite force acting on a body, in such a way that the lines of action of the two forces are not in the same straight line.



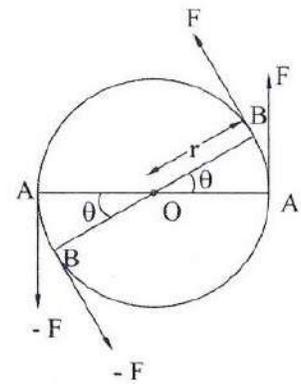
Explanation: These two forces form a couple and the moment of the couple about A is  $M_A$  and about B is  $M_B$ , then we can write.

$$\text{Couple } M_A = M_B = p \times d = Q \times d$$

(iii) Torque:

Torque is the rotating force and is equal to the moment of the couple. Torque is the product of one of the forces forming couple and perpendicular distance between the two opposite forces.

Explanation: Let us consider two equal and opposite forces F and -F acting tangentially on the rim of the circular disc from the extremities of diameter as shown in figure.



It forms a couple. If the disc rotates through a small angle  $\theta$ , the distance moved by the force F in rotating the body through an angle  $\theta$  = length of the arc AB =  $r\theta$ .

Here the length of the arc AB =  $r\theta$

Work done by a single force =  $F r \theta$

Work done by two forces constituting a couple =  $2 F r \theta$

Here  $2 F r \theta$  is the moment of the couple (or) torque ( $\tau$ )

Work done by the torque =  $\tau \theta$

1.11. TWISTING COUPLE ON A WIRE (Expression for Torque per Unit Twist)

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Consider a cylindrical wire of length  $l$  and radius  $r$  fixed at one end. It is twisted through an angle by applying couple to its lower end. Now the wire is said to be under torsion.

The wire can be assumed to be made up of a number of hollow cylindrical tubes (co-axial) whose radii varies from 0 to  $r$ .

Let us consider one such cylinder of radius  $x$  and thickness  $dx$  as shown in figure.

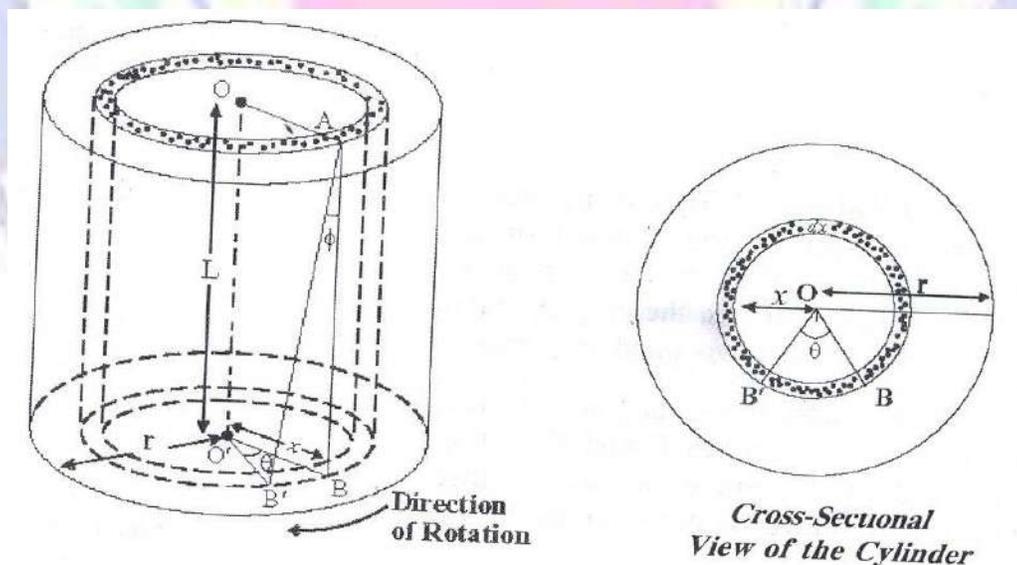
Due to the twisting torque the line  $AB$  which is initially parallel to the axis  $OO'$  of the cylinder is displaced to a position  $AB'$  through an angle  $\phi$  as shown in fig

The result of twisting the cylinder a shearing strain.

$$\text{The angle of shear} = BAB' = \phi$$

$$\text{Here } BB' = x \theta = L\phi$$

$$\phi = \frac{x \theta}{L} \quad \text{--- (1)}$$



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$$\text{Rigidity modulus } n = \frac{\text{Tangential stress}}{\text{Shearing strain}} = \frac{\text{Shearing stress}}{\text{Angle of Shear}(\Phi)}$$

$$\text{Shearing stress} = n\Phi \text{ --- (2)}$$

Substituting equation (1) in equation (2) we have

$$\text{Shearing stress} = \frac{n x \theta}{L} \text{ --- (3)}$$

$$\text{We know shearing stress} = \frac{\text{Shearing force}}{\text{Area}}$$

$\text{Shearing Force} = \text{Shearing stress} \times \text{Area on which the shearing force is acting}$

$$F = \frac{n x \theta}{L} 2\pi x dx \text{ --- (4)}$$

Moment of the force about the OO' axis of the cylinder = shearing force  $\times$  Distance

$$\text{Moment of the force} = \frac{n x \theta}{L} 2\pi x dx \cdot x$$

$$\text{Moment of the force} = \frac{2\pi n \theta}{L} x^3 dx \text{ --- (5)}$$

Twisting couple of the whole wire can be derived by integrating eqn.(5) within the limits 0 to r (Since the radii varied from 0 to r)

Twisting couple on the wire

$$C = \int_0^r \frac{2\pi n \theta}{L} x^3 dx$$

$$C = \frac{2\pi n \theta}{L} \left( \frac{r^4}{4} \right)$$

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$$C = \frac{\pi n \theta r^4}{2L} \quad \text{---> (6)}$$

If the twist  $\theta$  is unity i.e.  $\theta = 1 \text{ radian}$  then we can write

$$\text{The torque per unit twist } C = \frac{\pi n r^4}{2L} \quad \text{---> (7)}$$

Hollow cylinder

For a hollow cylinder of the same length  $L$  and inner radius  $r_1$  and outer radius  $r_2$

The torque per unit twist

$$C' = \int_{r_1}^{r_2} \frac{2\pi n \theta}{L} x^3 dx$$

$$= \frac{2\pi n \theta}{L} \left( \frac{r^4}{4} \right)_{r_1}^{r_2}$$

$$C' = \frac{\pi n \theta}{2L} (r_2^4 - r_1^4) \quad \text{---> (8)}$$

Twisting couple per unit twist of the cylinder

$$C' = \frac{\pi n}{2L} (r_2^4 - r_1^4) \quad \text{If } \theta = 1 \text{ radian}$$

### 1.12. Torsion Pendulum (Theory)

A circular metallic disc suspended using a thin wire that executes torsional oscillation is called torsion pendulum.

*Principle: When a disc (torsion pendulum) is rotated in a horizontal plane, the disc executes simple harmonic oscillation due to the restoring couple produced in the wire.*

It executes torsional oscillations whereas a simple pendulum executes linear oscillations

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### Description

A torsion pendulum consists of a wire with one end fixed to a split chuck and the other end fixed to the centre of the circular disc of radius  $R$  as shown in fig. Let  $L$  be the distance between the chuck end to the disc and  $r$  be the radius of the wire.

### Working

The circular disc is rotated in horizontal plane so the wire is twisted through an angle  $\theta$  thereby creating a restoring couple ( $C$ ) in the wire.

$$\text{Restoring couple} = C\theta \quad \text{---> (1)}$$

If the disk is released, it oscillates with angular velocity ' $\omega$ ' in the horizontal plane about the axis of the wire. These oscillations are known as torsional oscillations.

$$\text{The angular velocity } \omega = \frac{d\theta}{dt} \quad \text{---> (2)}$$

$$\text{Angular Acceleration } \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \quad \text{---> (3)}$$

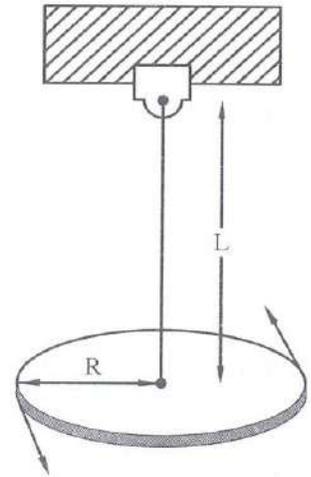
Here  $I$  is the moment of inertia of the circular disc, and  $\frac{d^2\theta}{dt^2}$  is the angular acceleration produced in the disc.

$$\text{Applied couple} = I \frac{d^2\theta}{dt^2} \quad \text{---> (4)}$$

In the equilibrium condition

Applied couple = Restoring couple

$$I \frac{d^2\theta}{dt^2} = C\theta$$



$$\frac{d^2\theta}{dt^2} = \frac{C\theta}{I} \quad \text{---> (5)}$$

This equation represents simple harmonic motion which shows that angular acceleration  $\frac{d^2\theta}{dt^2}$  is proportional to angular displacement and always directed towards the mean position.

*Period of Oscillation*

We know the time period of oscillation  $T = 2\pi \sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$

$$\text{Substituting from eqn. (5), we have } T = 2\pi \sqrt{\frac{\theta}{C\theta/I}}$$

$$\text{Time period of oscillation } T = 2\pi \sqrt{\frac{I}{C}} \quad \text{---> (6)}$$

$$\text{Frequency of oscillations } f = \frac{1}{2\pi} \sqrt{\frac{C}{I}} \quad \text{Hence } \frac{1}{T} = f$$

*Rigidity Modulus of the wire*

If r is the radius of the wire and L is the length of the wire suspended, then we know

$$\text{The torque per unit twist } C = \frac{n\pi r^4}{2L} \quad \text{---> (7)}$$

Substituting equation (7) in equation (6) We get

$$T = 2\pi \sqrt{\frac{I 2L}{n\pi r^4}}$$

$$T^2 = \frac{4\pi^2 2LI}{n\pi r^4}$$

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$$\text{Rigidity modulus of the wire } n = \frac{8\pi IL}{T^2 r^4} \quad \text{---> (8)}$$

Thus torsion pendulum is used to find the rigidity modulus for various materials

### 1.13. EXPERIMENTS USING TORSION PENDULUM - MOMENT OF INERTIA OF A REGULAR BODY

i) Determination of the moment of inertia of the disc and rigidity modulus of the wire using Torsion pendulum with mass

To determine the moment of inertia of the disc and rigidity modulus of the wire, the disc is set into torsional oscillations without any mass over it and the time period of oscillations (T) is measured.

The period of oscillations without mass  $T_o = 2\pi \sqrt{\frac{I}{C}}$

$$T_o^2 = \frac{4\pi^2 I}{C} \quad \text{---> (1)}$$

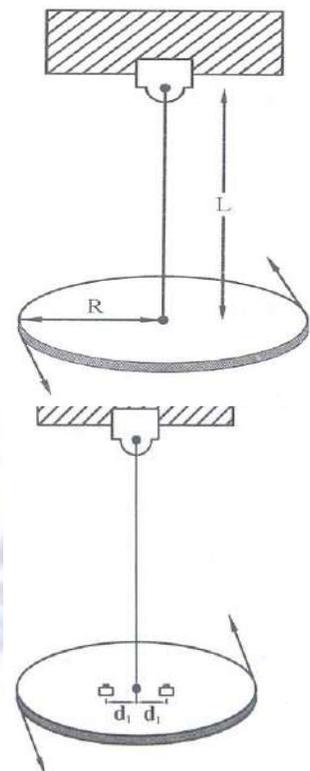
Where, I is the moment of inertia of the disc about the axis of rotation and C is the restoring couple.

The period of oscillation when the masses are at a distance  $d_1$

$$T_1 = 2\pi \sqrt{\frac{I_1}{C}}$$

$$T_1^2 = \frac{4\pi^2 I_1}{C} \quad \text{---> (2)}$$

Where,  $I_1$  is the moment of inertia of the disc along with two cylindrical masses placed over the disc at a distance  $d_1$

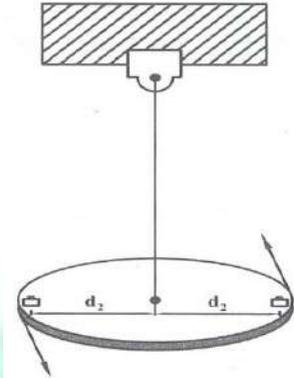


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The period of oscillation when the masses are at a distance  $d_2$

$$T_2 = 2\pi \sqrt{\frac{I_2}{C}}$$

$$T_2^2 = \frac{4\pi^2 I_2}{C} \quad \text{---> (3)}$$



Where,  $I_2$  is the moment of inertia of the disc along with two cylindrical masses placed over the disc at a distance  $d_2$

*Moment of Inertia of the disc*

$$\frac{T_o^2}{(T_2^2 - T_1^2)} = \frac{\frac{4\pi^2}{C} I}{\frac{4\pi^2}{C} (I_2 - I_1)}$$

$$\frac{T_o^2}{(T_2^2 - T_1^2)} = \frac{I}{(I_2 - I_1)}$$

$$I = \frac{(I_2 - I_1) T_o^2}{(T_2^2 - T_1^2)} \quad \text{---> (4)}$$

From the parallel axis theorem we can write the moment of inertia  $I_1$

$$I_1 = I + 2 I_m + 2m d_1^2 \quad \text{---> (5)}$$

Where

$I_m$  - Moment of inertia of each cylindrical mass passing through its centre.  $m$  - Mass of the cylindrical weights placed over the disc.

Similarly from the parallel axis theorem, we can write

$$I_2 = I + 2 I_m + 2m d_2^2 \quad \text{---> (6)}$$

From equation (5) and (6) we can write

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$$I_2 - I_1 = 2m (d_2^2 - d_1^2)$$

Moment of inertia of the disc about the axis of (wire) rotation

$$I = \frac{2m (d_2^2 - d_1^2) T_o^2}{(T_2^2 - T_1^2)} \quad \text{---> (7)}$$

b) Rigidity modulus of the wire

From the theory of torsion pendulum we know

$$\text{The Rigidity modulus of the wire } n = \frac{8\pi L}{T^2 r^4} \quad \text{---> (8)}$$

Substituting equation (8) in (9) we have

$$n = \frac{8\pi L}{T^2 r^4} \cdot \frac{2m (d_2^2 - d_1^2) T_o^2}{(T_2^2 - T_1^2)}$$

$$n = \frac{16\pi m L (d_2^2 - d_1^2) T_o^2}{(T_2^2 - T_1^2) r^4} \quad \text{---> (9)}$$

Equation (9) represents the Rigidity modulus of the wire.

#### 1.14. BENDING MOMENT (BENDING OF BEAMS)

Beam

A beam is defined as a rod (or) bar of uniform cross section whose length is very much greater than its other dimensions, such as breadth and thickness. It is commonly used in the construction of bridges to support roofs of the buildings etc., Since the length of the beam is much greater than its other dimensions the shearing stresses are very small.

Assumptions:

- (i) The length of the beam should be large.
- (ii) The load applied should be large.

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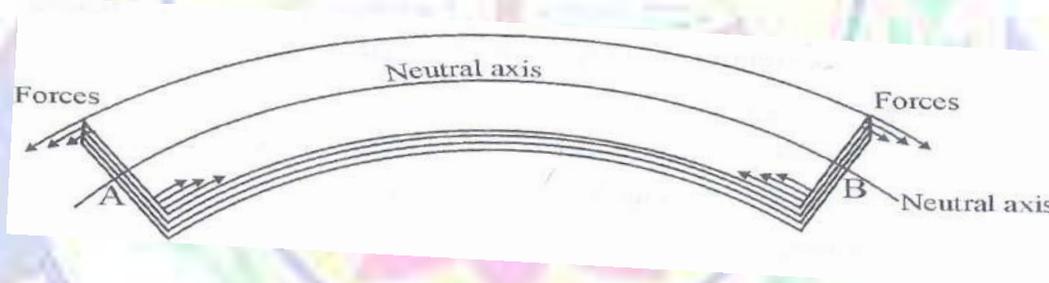
- (iii) The cross section of the beam remains constant.
- (iv) The shearing stresses are negligible.
- (v) The curvature of the beam is very small.

### 1.15. BENDING OF A BEAM AND NEUTRAL AXIS

Let us consider a beam of uniform rectangular cross section. This beam is made up of a large number of thin plane layers one above the other. Consider a beam which is bent an arc by the application of a load.



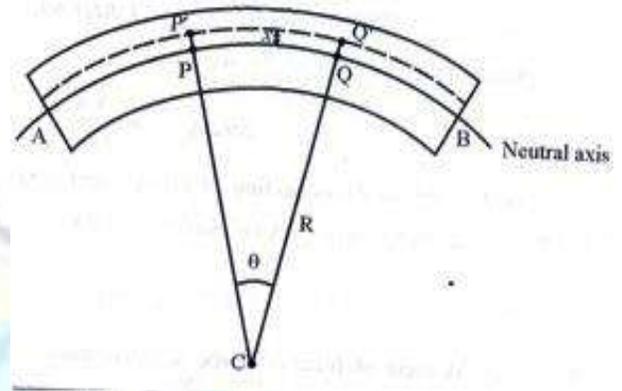
Taking a longitudinal section ABCD of the bent beam, the layers in the upper half are elongated while those in the lower half are compressed.



Let us consider a filament AB at the centre of the beam, which is remain unaltered is taken as reference axis called as neutral axis and the plane is called as neutral plane.

#### 1.15.1. EXPRESSION FOR THE BENDING MOMENT OF BEAM

Let us consider a beam under the action of deforming forces. The beam bends into a circular arc. Let AB be the neutral axis of the beam. The filament AB remains unchanged. Let PQ be the arc chosen from the neutral axis.



If  $R$  --- radius of curvature

$\theta$  --- Angle

Then we can write the original length  $PQ = R\theta$  --- (1)

Let us consider a filament  $P'Q'$  at a distance ' $x$ ' from the neutral axis.

$\therefore$  We can write the extended length  $= P'Q' = (R + x)\theta$  --- (2)

From eqn.(1) and eqn.(2) we have

Increase in its length  $= P'Q' - PQ$

Increase in its length  $= (R + x)\theta - R\theta$

$\therefore$  Increase in its length  $= x\theta$  --- (3)

We know that  $\text{Linear strain} = \frac{\text{increase in length}}{\text{original length}}$

$$\text{Linear strain} = \frac{x\theta}{R\theta}$$

$$\text{Linear strain} = \frac{x}{R} \text{ --- (4)}$$

$$\text{The young's modulus of the beam } Y = \frac{\text{Linear stress}}{\text{Linear strain}}$$

Sub. Eqn. (4) in eqn. (5), we have

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$$\text{Linear stress} = \frac{Yx}{R}$$

If  $\delta A$  is the area of cross section of the filament  $P'Q'$ , then the force on the area  $\delta A = \text{stress} \times \text{Area}$

$$\text{Force} = \frac{Yx}{R} \delta A$$

Therefore moment of force about neutral axis PQ is given by

$$\text{Moment of force} = \text{Force} \times \text{Perpendicular distance}$$

$$= \frac{Yx}{R} \delta A \times (x) = \frac{Y}{R} \delta A x^2$$

$\therefore$  The sum of moment forces acting on all layers about the neutral axis

$$= \frac{Y}{R} \Sigma x^2 \delta A$$

Where  $I = \Sigma x^2 \delta A = AK^2$  is called geometrical moment of inertia.

$\therefore$  Total moment of all the forces (or) Internal bending moment

$$= \frac{YI}{R} \quad \text{---> (5)}$$

Special cases

Rectangular cross section:

If 'b' the breadth and 'd' is the thickness of the beam, then  $I = \frac{bd^3}{12}$  Sub. The value of eqn.(5) we can write

Bending moment for a rectangular cross section

$$= \frac{Ybd^3}{12R} \quad \text{---> (6)}$$

Circular cross section:

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If 'r' is the radius of the circular cross section then area  $A = \pi r^2$  and  $I = \frac{\pi r^4}{4}$  ; Sub.

The value of I eqn. (5) we can write

Bending moment for a circular cross section

$$= \frac{\pi Y r^4}{4R} \quad \text{---> (7)}$$

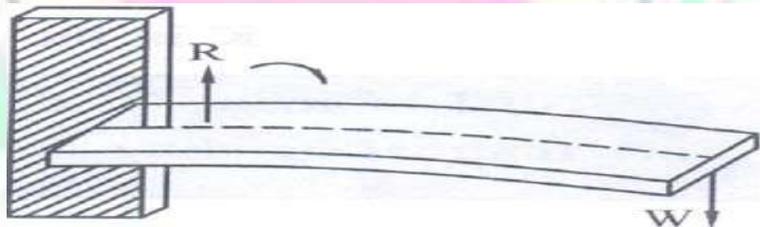
## 1.16. CANTILEVER

Cantilever

A cantilever is a beam fixed horizontally at one end and loaded at the other end. This type of bending is known as cantilever.

Theory

Let us consider a beam at one-end and loaded at its other free end. Due to the load applied at the free end, a couple is created between the two forces. (i.e)



- (i). Force (load 'W') applied at the free end towards downward direction and
- (ii). Reaction (R) acting in the upward direction at the supporting end.

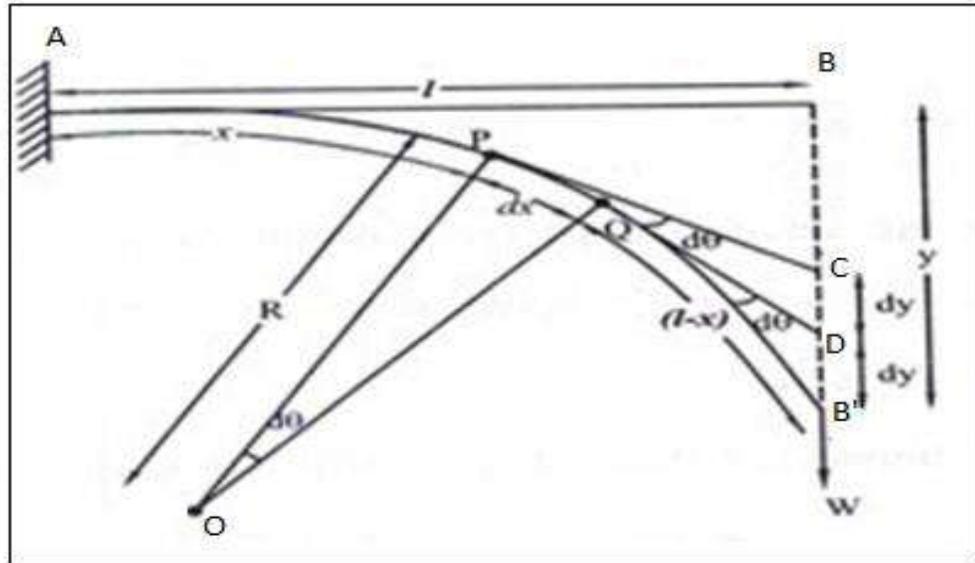
## 1.17. DERIVATION FOR DEPRESSION OF A CANTILEVER - LOADED

AT ITS ENDS

Theory

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Let ' $l$ ' be the length of the cantilever AB fixed at 'A'. Let ' $W$ ' be the weight suspended (loaded) at the free end of the cantilever. Due to the load applied the cantilever moves to a new position 'BB''.



Let us consider the section of cantilever P at a distance  $x$  from the fixed end A. It is a distance  $(l-x)$  from the loaded end B'

$$\therefore \text{The external bending moment} = W \times PB' = W \times (l-x) \quad \text{---} \rightarrow (1)$$

$$\text{We know the internal bending moment} = \frac{YI}{R} \quad \text{---} \rightarrow (2)$$

Under equilibrium condition,

$$\text{External bending moment} = \text{Internal bending moment}$$

$$W \times (l-x) = \frac{YI}{R} \quad \text{---} \rightarrow (3)$$

Let Q is the other point at a distance  $dx$  from P

$$\text{The arc length } PQ = dx$$

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Let 'O' be the centre of curvature of the element PQ and let 'R' be the radius of curvature.

The angle  $\angle POQ = d\theta$

$$dx = R d\theta \quad \text{---> (4)}$$

The tangents are drawn at P and Q meeting the BB' at C and D

$$d\theta = \frac{dx}{R} \quad \text{---> (5)}$$

Vertical depression  $CD = dy = (l - x)d\theta \quad \text{---> (6)}$

From eqns. (4) and (5)

$$\frac{dx}{dy} = \frac{R d\theta}{(l - x)d\theta}$$

$$R = (l - x) \frac{dx}{dy} \quad \text{---> (5)}$$

Substitute equation (5) in equation (3)

$$W \times (l - x) = \frac{YI}{(l - x) \frac{dx}{dy}} \quad \text{---> (6)}$$

$$W \times (l - x)^2 dx = YI dy$$

$$dy = \frac{W}{YI} \times (l - x)^2 dx$$

$\therefore$  Total depression at the free end of the cantilever can be derived by integrating within the limits '0' to 'l'

$$y = \frac{W}{YI} \int_0^l (l - x)^2 dx$$

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$$y = \frac{W}{YI_g} \int_0^l (l^2 - 2lx + x^2) dx$$

$$y = \frac{W}{YI_g} \int_0^l (l^2 x - \frac{2lx^2}{2} + \frac{x^3}{3})$$

$$y = \frac{W}{YI_g} (l^3 - l^3 + \frac{l^3}{3})$$

$$y = \frac{W l^3}{3YI_g}$$

Therefore depression of the cantilever at free end

$$y = \frac{Wl^3}{3YI_g} \quad \text{---> (8)}$$

*Determination of young's modulus of the Cantilever*

The young's modulus of the beam is determined by using the depression produced in the cantilever

$$y = \frac{Wl^3}{3YI_g}$$

The young's modulus of the beam  $Y = \frac{Wl^3}{3yI_g}$

Special cases

(i) Rectangular cross section:

If 'b' the breadth and 'd' is the thickness of the beam then  $I = \frac{bd^3}{12}$ , and the weight  $W=Mg$ , where M is the mass suspended at the free end and g is acceleration due to gravity.

Substituting this value in equation (5)

Depression of the cantilever at free end  $y = \frac{Wl^3}{3YI_g}$

The young's modulus of the beam  $Y = \frac{Wl^3}{3yI_g}$

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$$Y = \frac{Mg l^3}{3y \left( \frac{bd^3}{12} \right)}$$

The young's modulus of the beam  $Y = \frac{4Mg l^3}{bd^3 y}$

(ii) Circular cross section:

If 'r' is the radius of the circular cross section then,  $I = \frac{\pi r^4}{4}$

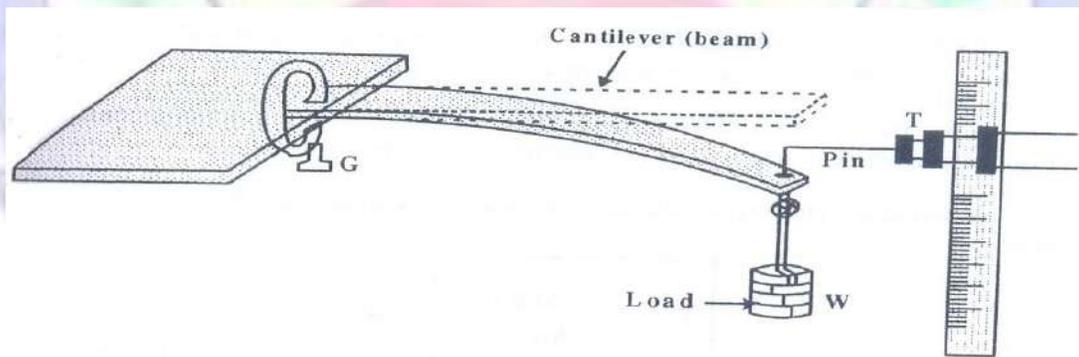
Depression of the cantilever at free end  $y = \frac{wl^3}{3YI}$

The young's modulus of the beam  $Y = \frac{Mgl^3}{3y \left( \frac{\pi r^4}{4} \right)} = \frac{4Mgl^3}{3\pi r^4 y}$

### 1.18. Experimental determination of young's modulus of the

#### Cantilever

The given bar is fixed rigidly at one end and weight hanger is suspended at the other end. A pin is fixed vertically at the free end of the beam. A travelling microscope (T) is focused on the pin.



The microscope is adjusted in such a way that horizontal cross wire coincides with the tip of the pin. The initial reading in the microscope on vertical scale is noted. A suitable mass M is placed on the hanger. The reading in the microscope is again noted. The difference between two readings of microscope gives the depression y corresponding to load M. Now the weight is added in steps of 50grams. Each time the tip of the pin is made to touch the horizontal cross wire and the readings are noted from the vertical scale

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of the microscope. Then, the experiment is also repeated by decreasing the weights. The observation are tabulated as follows

S.No	Load (M)	Microscope Readings			Mean depression y for a load of M Kg
		Increasing Load	Decreasing Load	Mean	
Unit	$10^{-3}\text{Kg}$	$10^{-2}\text{m}$	$10^{-2}\text{m}$	$10^{-2}\text{m}$	$10^{-2}\text{m}$
	W				
1	W+50				
2	W+100				
3	W+150				
4	W+200				
5	W+250				
				Mean y	

From these observations mean depression y corresponding to each value of M is obtained. The length  $l$  of the beam  $a$ , its breadth  $b$  (by vernier calipers) and the thickness  $d$  (by the screw gauge) are measured.

*The young's modulus of the beam by Cantilever method is determined by using the relation*

$$Y = \frac{4Mg l^3}{bd^3y}$$

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