Newton's Law of impact :-

Newton described the elasticity of the collision by considering the velocities of approach and recession

$(A) \xrightarrow{u}$ $\qquad$ $(B) \xrightarrow{u'}$

$(A) \xrightarrow{v}$ $\qquad$ $(B) \xrightarrow{v'}$

Velocity of approach $= u - u'$

Velocity of recession $= v' - v$.

defined a quantity known as the co-efficient of restitution. (e)

$$\text{co-efficient of resititution (e)} = \frac{(v' - v)}{(u' - u)}$$

This is known as newton's Law of impact the ratio $\frac{v' - v}{u' - u}$ must always be less than one except for a perfectly elastic collision where it is equal to 1.

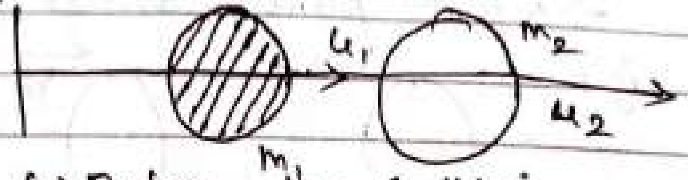
for to perfectly inelastic, collision $e = 0$. This masses that is (e) perfectly elastic collision the velocity of approach of the two bodies before the impact is equal to their velocity of separation an after the impact.

Coefficient of Restitution :-

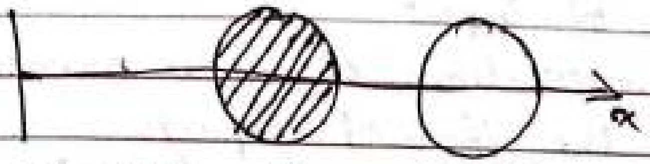
A common method of describing an

inelastic collision is with the coefficient of restitution (e) is defined by.

$$e = \frac{V_f}{V_i} = \frac{\text{relative velocity after}}{\text{relative velocity before}}.$$



(a) Before the Collision



(b) after the collision

Consider a heat on collision b/w two masses m_1 and m_2 moving with Velocities u_1 and u_2 before the Collision. Let V_1 and V_2 be the velocities the collision. The relative velocity of approach before

$$= u_1 - u_2 = -(V_{rel})_i$$

After the collision $= V_2 - V_1 = (V_{rel})_f$

e = Coefficient of restitution.

$$e = \left| \frac{V_2 - V_1}{u_2 - u_1} \right|$$

'e' Varies between 0 and 1 "e' for wood 0.4 – 0.6

Glass 0.93 – 0.95

steel 0.5 – 0.8

lead 0.12 – 0.18

Impact Of a Smoth sphere on a fixed place :–

Impact occurs when two body's colied a very short time period large impulse force to be exerted between the body.

(ex) ⇒ Impact are a hamber straight in nail

⇒ a bat straicking a ball

⇒ The line of impact is the line throug the mass centre of colied on partial.

⇒ In genteal two type of Impacts.

→ Central Impact

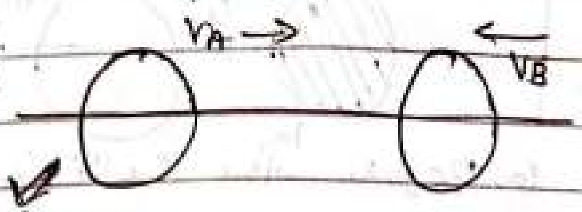
→ Oblique Impact.

Central Impact :-

⇒ It occurs when the direct of motion of the two colied of the particules are alone the line of impact.

⇒ Central impact happens, when velocity of the two object are alone of the line impact.

⇒ once the particles contact the may deform non-rigit.

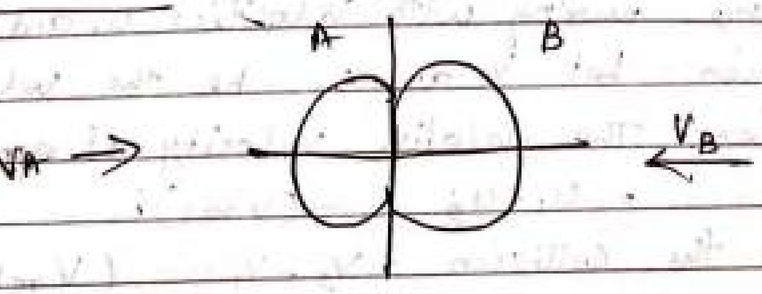


Line of impact

The Conservations of linear momentum applied along the line of impact.

$$(m_A V_A) + (m_B V_B) = (m_A V_A)_2 + (m_B V_B)_2$$

Oblique impact :-



⇒ One ~~or~~ both of the particles motion is at on the angle two the line impact typecally there will be four unknowns Magnitude and direction, and final velocity.

Conservation of momentum and the co-efficient of restitution equation are applied the line of impact (x-axis).

$$(m_A V_{Ax})_1 + (m_B V_{Bx})_1 = (m_A V_{Ax})_2 + (m_B V_{Bx})_2$$

$$e = \frac{(V_{Bx})_2 - (V_{Ax})_2}{(V_{Ax})_1 - (V_{Bx})_1}$$

Momentum of each particles conserved in the direct perpenticular to the line of impact (y-axis)

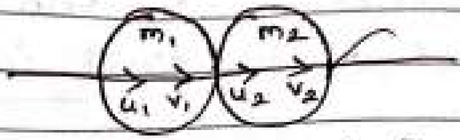
$$(m_A V_{Ay})_1 = (m_A V_{Ax})_2$$

$$(m_B V_{By})_1 = (m_B V_{By})_2$$

Direct impact b/w two smooth sphere :-

Statement :-

Impact between two smooth sphere is set to be direct if the direction of motion of each smooth sphere before the impact is along the normal at the point of contact.



⇒ A smooth sphere are mass(m_1) moving with the velocity u_1, impinging on another smooth sphere of mass (m_2) moving in the same direction with velocity u_2.

⇒ If e is co-efficient of restitution b/w them.

⇒ Let v_1 and v_2 be the velocities of two sphere along the common normal after impact.

By the principle of conservation of momentum

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2 \quad \text{--- (1)}$$

By Newton's experiments law

$$v_1 - v_2 = -e(u_1 - u_2) \quad \text{--- (2)}$$

multiplying by m_2 equ (2) and add equ (1)

$$m_2(v_1 - v_2) = -e\, m_2 (u_1 - u_2) \quad \text{--- (3)}$$

$$v_1(m_1 + m_2) = m_2 u_2 (1+e) + u_1(m_1 - e m_2) \quad \text{--- (4)}$$

$$v_1 = \frac{m_2 u_2 (1+e) + u_1 (m_2 - e m_2)}{m_1 + m_2} \quad \text{--- (5)}$$

multiplying equ (2) by m_1 and sub equ (1).

$$v_2(m_1 + m_2) = m_1 u_1 (1+e) + u_2(m_2 - e m_1) \quad \text{--- (6)}$$

$$v_2 = \frac{m_1 u_1 (1+e) + u_2 (m_2 - e m_1)}{m_1 + m_2} \quad \text{--- (7)}$$

equ (5) & (7) give the velocities of two smooth spheres after velocities.

Cor 1 :- The impulse of the sphere of mass m_1 = change of momentum.

$$m_1(v_1 - v_2) = \frac{m_1 m_2 (1-e)(u_2 - u_1)}{m_1 + m_2}$$

This is equal and Opposite to the impulse on the Sphere mass m_2.

Cor. 2 :- If $e = 1$ and $m_1 = m_2$ then $v_1 = u_2$, $v_2 = u_1$.

If two equal perfectally elastic Sphere impaining directly inter change the velocity.

Loss of K.E due to direct impact of two smooth Sphere :-

Statement :- m_1, m_2 be the mass u_1, u_2, v_1 and v_2 to the velocity before and after impact and e the coefficient of restitution.

The principle of conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \longrightarrow (1)$$

by Newton Experiment Law

$$(v_1 - v_2) = -e(u_1 - u_2) \longrightarrow (2)$$

Square of both equations, and Multiple the squre of the second equation by m_1 and m_2 and add the result :

$$m_1^2 v_1^2 + m_2^2 v_2^2 = m_1^2 u_1^2 + m_2^2 u_2^2 \quad (3)$$

$$m_1 m_2 (v_1 - v_2)^2 = (-e)^2 m_1 m_2 (u_1 - u_2)^2 \quad (4)$$

$$m_1^2 v_1^2 + m_2^2 v_2^2 + m_1 m_2 v_1^2 - m_1 m_2 v_2^2 = m_1^2 u_1^2 + m_2^2 u_2^2 + e^2 m_1 m_2 (u_1^2 - 2 m_1 m_2 u_2^2)$$

$$(m_1^2 + m_1 m_2) v_1^2 + (m_2^2 - m_1 m_2) v_2^2 = (m_1 u_1 + m_2 u_2)^2 + m_1 m_2 (u_1^2 - u_2^2) + e^2 m_1 m_2 (u_1 - u_2)^2$$

$$v_1^2 m_1 (m_1 + m_2) + m_2 v_2 (m_2 - m_1) = (m_1 + m_2)(m_1 u_1^2 + m_2 u_2^2) - m_1 m_2 (u_1 - u_2)(1 - e^2)$$

multiply by $1/2$

$$\tfrac{1}{2} v_1^2 m_1 (m_1 + m_2) + \tfrac{1}{2} m_2 v_2 (m_2 - m_1) = \tfrac{1}{2}(m_1 + m_2)(m_1 u_1^2 + m_2 u_2^2) - \tfrac{1}{2} m_1 m_2 (u_1 - u_2)(1 - e^2)$$

$$\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v^2 = \tfrac{1}{2} m_1 u_1^2 + \tfrac{1}{2} m_2 u_2^2 - \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2 (1 - e^2)$$

$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \rightarrow$ After K.E impact

$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \rightarrow$ Before K.E impact

Note :- When $\varepsilon = 1$ the loss of K.E is 0.

$\varepsilon < 1$ so that $(1-e)$ possitive $(u_1 - u_2)$ is also positive. This is always a loss of K.E due to impact.

The K.E lost during impact is converted (i) Sound (ii) heat (iii) Vibration or rotation of the colaital body.

When $\varepsilon = 0$ the Loss of K.E is

$$K.E = \frac{1}{2} \frac{m_1 m_2 (u_1 - u_2)^2}{m_1 + m_2}$$

That is maximum K.E impact of Plastic body.