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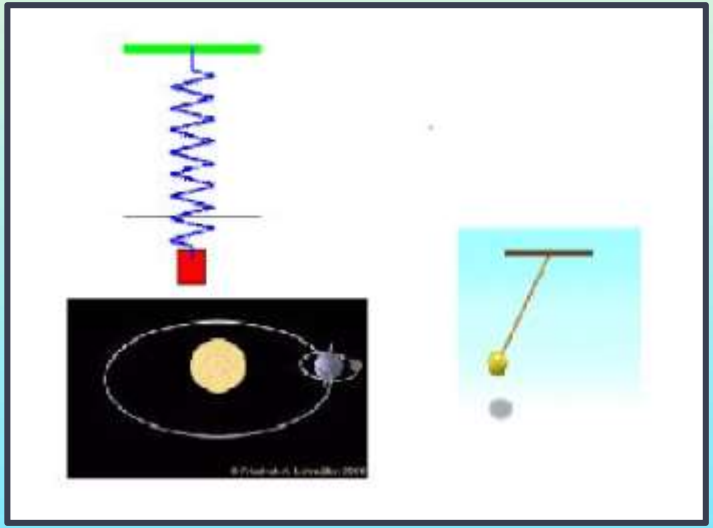
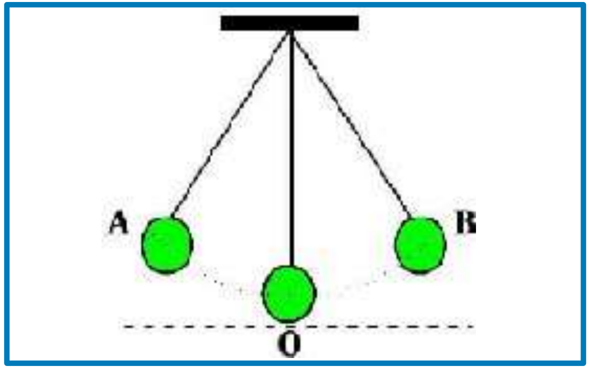
Oscillations



Presented by
Dr.S.karthika Rani
Assistant Professor
Department of Physics
CPA College, Bodi.

Periodic motion

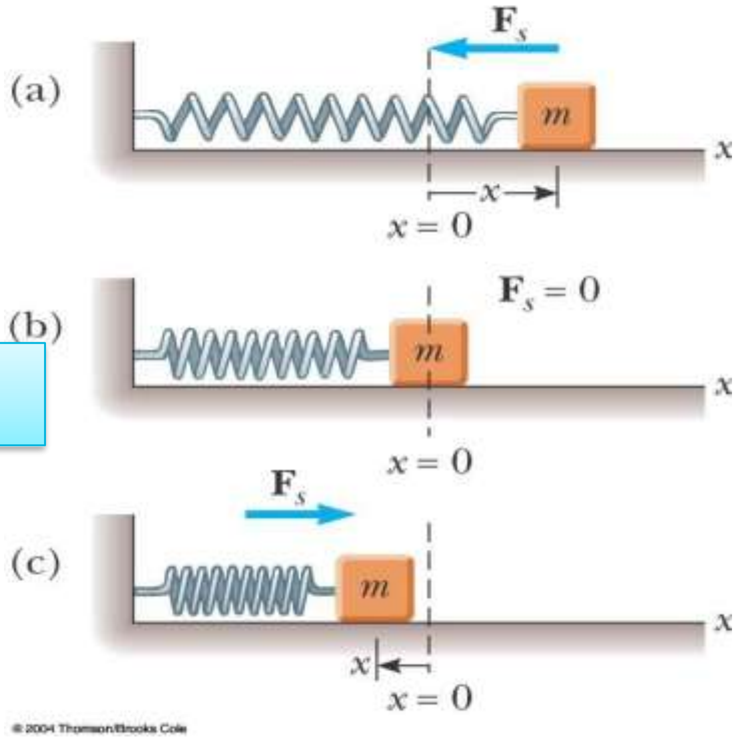
- Motion of an object that regularly repeats
- The object returns to a given position after a fixed time interval
- A special kind of periodic motion occurs in mechanical systems when the force acting on the object is proportional to the position of the object relative to some equilibrium position



Linear Simple Harmonic Motion(S.H.M)

- The linear periodic motion of a body, in which force is always directed towards the mean positon and its magnitude is proportional to the displacement from the mean position.
- The force is directly proportional to the displacement but its direction is opposite to that displacement.
- That is, $Fs = -kx$

$Fs \propto x$



Differential Equation of S.H.M

- 1. By definition of linear S.H.M, the force acting on a particle is given by

  ***F* = -kx** …...................(1)

- The acceleration of particle is given by, $$a = \frac{dv}{dt} = \frac{d\left(\frac{dx}{dt}\right)}{dt} = \frac{d^2x}{dt^2}$$
- According to Newton's second law of motion

$$F = ma$$

$$\therefore F = m\left(\frac{d^2x}{dt^2}\right) \text{ ....(2)}$$

- From equation (1) and (2),

$$m\left(\frac{d^2x}{dt^2}\right) = -kx$$

$$\therefore \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\therefore \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \text{....................(3)}$$

$$\text{where, } \frac{k}{m} = \omega^2 = contsant$$

$$\therefore \frac{d^2x}{dt^2} + \omega^2 x = 0 \text{....................(4)}$$

Acceleration(a) of S.H.M.

- Acceleration is velocity per unit time. We can calculate the acceleration of a particle performing S.H.M.
- The differential equation of S.H.M.

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

Let $\frac{k}{m} = \omega^2$, a constant

$$\therefore \frac{d^2x}{dt^2} + \omega^2 x = 0$$

$\therefore$ Acceleration, a = $\frac{d^2x}{dt^2} = -\omega^2 x$ ....(1).

- The (-ve) sign shows that acceleration and displacement have opposite direction

Velocity(V) of S.H.M.

- Velocity is distance per unit time. Velocity of particle is $v = \frac{dx}{dt}$
- We can obtain the expression for velocity using the expression for acceleration

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v\frac{dv}{dx}$$

- As defined acceleration in previous side **a** = $-\omega^2 x$ ……(1)
- Hence equation(1) is $v\frac{dv}{dx} = -\omega^2 x$

$$\therefore vdv = -\omega^2 x \, dx$$

- Integrating both sides

$$\frac{v^2}{2} = \frac{-\omega^2 x^2}{2} + C$$

At an extreme position (a turning point of the motion), the velocity of the particle is zero. Thus, v = 0 when x = ±A, where A is the amplitude.

$$\therefore 0 = \frac{-\omega^2 A^2}{2} + C \therefore C = \frac{\omega^2 A^2}{2}$$

$$\therefore \frac{v^2}{2} = \frac{-\omega^2 x^2}{2} + \frac{\omega^2 A^2}{2}$$

$$\therefore v^2 = \omega^2(A^2 - x^2)$$

$$\therefore v = \pm\omega\sqrt{A^2 - X^2} \text{ ......(2)}$$

Displacement of S.H.M

- Since $v = dx/dt$
- We can write equation (2) as

$$\frac{dx}{dt} = \omega\sqrt{A^2 - x^2}$$

$$\therefore \frac{dx}{\sqrt{A^2 - x^2}} = \omega dt$$

$$\int \frac{dx}{\sqrt{A^2 - x^2}} = \int_0^t \omega dt$$

$$\sin^{-1}\left(\frac{x}{A}\right) = \omega t + x \quad ....(3)$$

$$\frac{x}{A} = \sin(\omega t + x)$$

- Displacement as a function of time is

$$x = A\sin(\omega t + x).$$

Period of S.H.M.(T)

- The time taken by the particle performing S.H.M. to complete one oscillation is called period of S.H.M.

  $T = 2\pi/\omega$ ….Particle completes one oscillation.

  $\omega^2 = k/m$ = force per unit displacement/mass &

  $T = 2\pi/\omega = 2\pi/\sqrt{(\omega^2)} = 2\pi/\sqrt{(k/m)}$

  $\mathbf{T = 2\pi \sqrt{(m/k)}}$

- ***Frequency*** (n) is defined as number of oscillations performed by particle performing S.H.M. per unit time is called frequency of S.H.M.

  $\mathbf{n = 1/T = \omega/ 2\pi = 1/2\pi\sqrt{(m/k)}}$

Phase in S.H.M.

- The quantity which describes the state of oscillation of particle performing S.H.M is called phase of S.H.M.
- Phase **ɵ = 0** indicates that particle is at mean position, moving towards positive, during the beginning of the first oscillation.

Phase angle ɵ = 360 or 2π is the beginning of second oscillation

- Phase **ɵ = 180 or π**, During its first oscillation, the particle is at mean position and moving to negative.
- Phase **ɵ = 90 or π/2,** then particle is at positive extream position during first oscillation for second oscillation it will be (360+90)
- Phase **ɵ = 270** indicates that particle is negative extreme postion during first oscillation. For second oscillation it will be ɵ = (360+270)

Simple Pendulum

An ideal simple pendulum is a heavy particle suspended by a massless, inextensible, flexible string from a rigid support.

A practical simple pendulum is a small heavy (dense) sphere (called bob) suspended by a light and inextensible string from a rigid support.

The distance between the point of suspension and center of gravity of the bob is called the length of the pendulum.

At extreme position the forced acting on the bob are.

- Tension T^{I} towards the support.
- Weight mg in vertically downward direction.

Weight mg can be resolved into two components –

i. mg $\cos\theta$ *along the string*

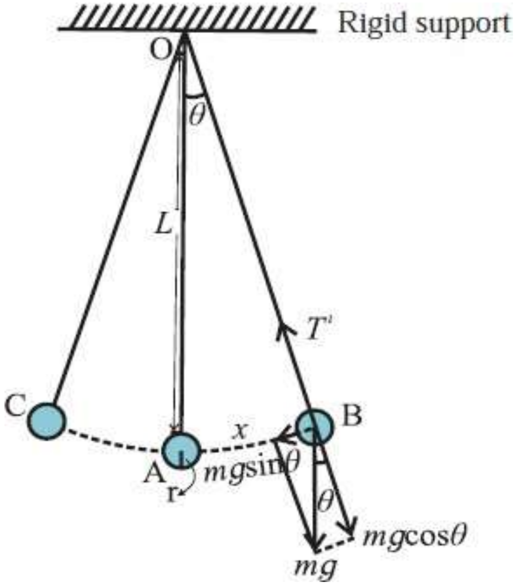
ii. mg $\sin\theta$ $\perp^{ar}$ *to the string*

$$\therefore Restoring\ force = F = -mg\sin\theta \ldots\ldots\ldots\ldots\ldots\ldots \text{(I)}$$

$$As\ \theta\ is\ very\ small, \sin\theta \cong \theta$$

$$\therefore F \cong mg\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(II)}$$

$$from\ fig., \quad \theta = \frac{x}{L}$$



$from\ fig.,\quad \Theta = \frac{x}{L}$

$\therefore F = -mg\frac{x}{L}$............................ (III)

$As\ m, g\ and\ L\ are\ const., F \propto -x, Hence\ the\ bob\ performs$

$S.H.M$

We know, f = ma

$\therefore ma = -\frac{mg}{L}$

$a = -g.\frac{x}{L}$

$\therefore \frac{a}{x} = \frac{-g}{L} = \frac{g}{L}$ (in magnitude) ....(IV)

$we\ know$

$$T = \frac{2\pi}{\sqrt{acc^n perunit\ displacement}}$$

$$T = \frac{2\pi}{\sqrt{\frac{g}{L}}}$$

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{.............................} (5)$$

Assumptions –

1) amplitude of oscillation should be very small

2) the length of the string should be large

3) the bob moves along a single vertical plane.

Frequency of oscillations (n) is,

$$n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \quad \text{..................................} (6)$$

Laws –

- $T \propto \sqrt{L}$
- $t \propto 1/\sqrt{g}$

- T does not depends on m
- For small amplitudes T does not depends on amplitude.

<u>Second’s pendulum</u>

A simple pendulum whose period is two seconds is called second’s pendulum.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$2 = 2\pi\sqrt{\frac{L_s}{g}}$$

$$\therefore Ls = \frac{g}{\pi^2}$$

Damped Oscillations

- Periodic oscillations of gradually decreasing amplitude are called damped harmonic oscillations and the oscillator is called damped harmonic oscillator.
- The damping force (Fd) is directly proportional to the speed (v) of the vane. and the block

$$F_d \propto -V$$

Where, b – damping constant.

$$F_d = -bV$$

The force on the block from the spring is,

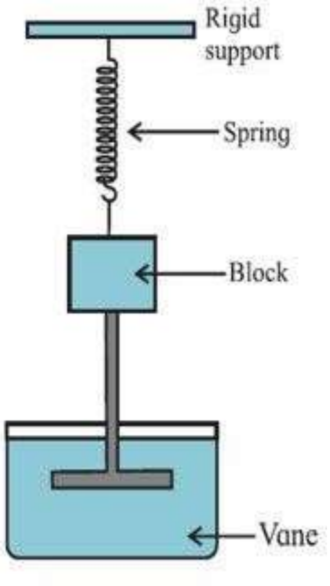
$$Fs = -kx$$

The total force is

$$F = Fd + Fs$$

$$ma = -bV - kx$$

$$ma + bV + kx = 0$$



$$m\frac{d^2x}{dt2} + b\frac{dx}{dt} + kx = 0 \ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

The solution is,

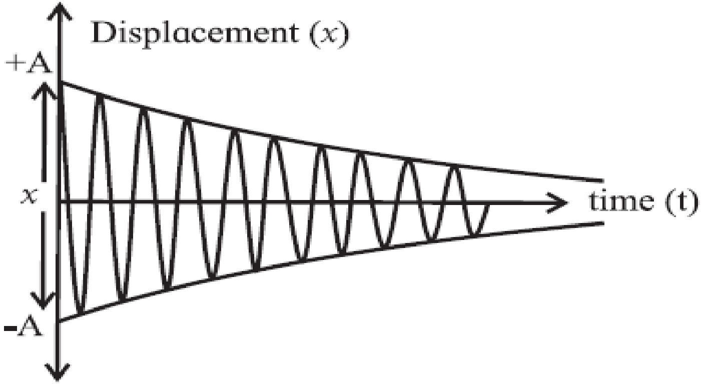
$$X = Ae^{-bt/2m}.\cos(\omega' t + \emptyset) \ldots\ldots\ldots\ldots\ldots\ldots \text{(II)}$$

Where

$Ae^{-bt/2m}$ – *amplitide of the damped harmonic oscillations.*

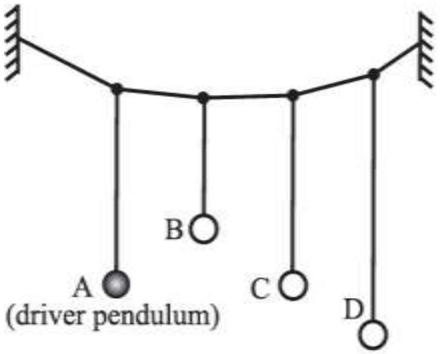
$$\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} \ldots\ldots\ldots\ldots\ldots\ldots (III)$$

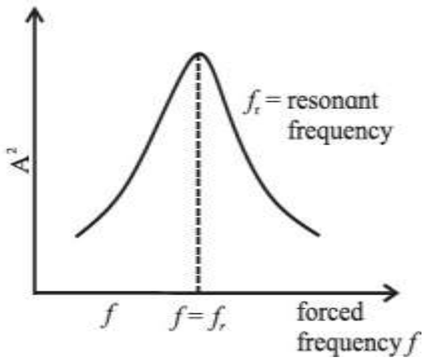
$$T = \frac{2\pi}{\omega'} = \frac{2\pi}{\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}} \ldots\ldots\ldots\ldots\ldots\ldots \text{(IV)}$$



Free Oscillations, Forced Oscillations And Resonance

- Free oscillations – Body oscillates with its natural frequency.
- Forced vibrations – Body oscillates with a frequency equal to frequency of driving force.
- Resonance – If the driving frequency becomes exactly equal to fundamental frequency of body, it vibrates with maximum amplitude.





THANK YOU