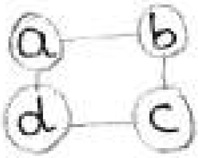
Representing Graphs and Graphs Isomorphism :

Adjacency lists :

can be used to represent a graph with no multiple edges

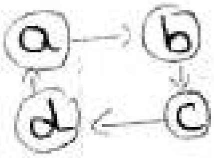
A table with 1 row per vertex, listing its adjacent vertices.



vertex	Adjacent vertex
a	b, d
b	a, c
c	b, d
d	a, c

Directed Adjacency lists :

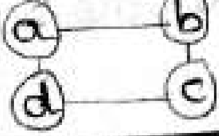
1 row per vertex, listing the terminal T.V vertices of each edge incident from the vertex.



Initial T.vertex	T.vertices
a	b
b	c
c	d
d	a

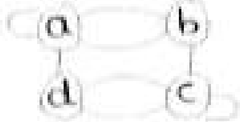
Adjacency matrix :

let the adjacency matrix $A_G = [a_{ij}]$ of a graph G is the $n \times n$ $(n = |V|)$ zero - one Matrix, where $a_{ij} = 1$ if $\{v_i, v_j\}$ is and edge of G, and is 0 otherwise



$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Can extend to graphs with loops and multiple edges by letting each matrix elements be the number of links (possibly >1) between the nodes.



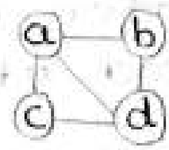
$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 2 & 0 \end{bmatrix}$$

Incidence matrices:

Let $G=(V,E)$ be an undirected graph with $V=\{v_1,\dots,v_n\}$ and $E=\{e_1,\dots e_m\}$. Then the incidence matrix with respect to this ordering of V and E is the $n \times m$ matrix $M=[m_{ij}]$ where $m_{ij}=1$ if e_j is incident with v_i, and is 0 otherwise.

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Example : 1

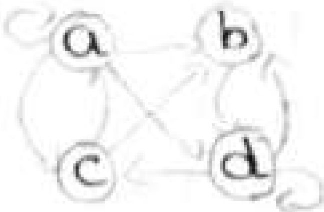


Vertex	Adjacent Vertices
a	b, c, d
b	a, d
c	a, d
d	a, b, c

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Example : 2

Use an adjacency list and adjacency matrix to represent the given graph.

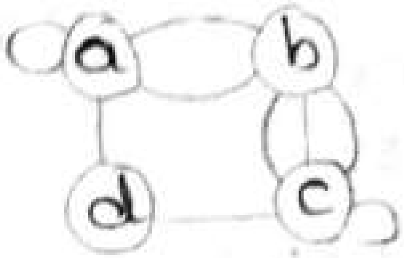


Initial vertex	Terminal vertex
a	a, b, c, d
b	d
c	a, b
d	b, c, d

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Example : 3 :

Draw an undirected graph represented by the given adjacency matrix.



$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$