Def :

A Recurrence Relation for the sequence $\{a_n\}$ is an equation that shows an in terms of one or more of the previous terms of the sequence $a_0, a_1, a_2 \cdots a_{n-1}$, for all integer n with $n \geq n_0$, where n_0, n_0 is a non-negative integer

Note:

* A sequence is called a solution of a recurrence relation if its terms satisfied the recurrence relation

Example

1) The Fibonancci sequence is defined by the recurrence relation $a_n = a_{n-2} + a_{n-1}$, $n \geq 2$, with the initial conditions $\boxed{a_0 = 1}$ and $\boxed{a_1 = 1}$

Example

2) Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-2} + a_{n-1}$ for $n = 2, 3, 4, 5 \ldots$ and suppose that $a_0 = 3$ and $a_1 = 5$ what are a_2, a_3, a_4 and a_5?

Sol:

Given

$$a_0 = 3, a_1 = 5$$

$$a_n = a_{n-2} + a_{n-1}$$

Put, $n = 2 \Rightarrow a_2 = a_{2-2} + a_{2-1}$

$$= a_0 + a_1$$

$$= 3 + 5$$

$$\boxed{a_2 = 8}$$

Put, $n=3 \Rightarrow a_3 = a_{3-2} + a_{3-1}$
$= a_1 + a_2$
$= 5+8$
$\boxed{a_3 = 13}$
Put, $n=4 \Rightarrow a_4 = a_{4-2} + a_{4-1}$
$= a_2 - a_3$
$= 8 + 13$
$\boxed{a_4 = 21}$
Put, $n=5 \Rightarrow a_5 = a_{5-2} + a_{5-1}$
$= a_3 + a_4$
$= 13 + 2$
$\boxed{a_5 = 34}$

3) find the first five terms of the sequence defined by each of these recurrence relations and initial conditions

a) $a_n = 6\, a_{n-1}$ given $a_0 = 2$
b) $a_n = a_{n-1} + 3a_{n-2}$, $a_0 = 1$, $a_1 = 2$
c) $a_n = a_{n-1} + a_{n-3}$, $a_0 = 1$, $a_1 = 2$, $a_2 = 0$

a) $a_n = 6\ a_{n-1}$ given $a_0 = 2$
Put $n=1 \Rightarrow a_1 = 6\, a_{1-1}$
$= 6a_0$
$= 6 \times 2$
$\boxed{a_1 = 12}$
Put $n=2 \Rightarrow a_2 = 6\ a_{2-1}$
$= 6a_1$
$= 6 \times 12$
$\boxed{a_2 = 42}$
$a_3 = 6a_{3-1}$
$= 6a_2$
$= 6 \times 72$
$\boxed{a_3 = 432}$

$a_4 = 6a_{4-1}$
$= 6a_3$
$= 6 \times 432$
$a_4 = 2592$

b) $a_n = a_{n-1} + 3a_{n-2}$, $a_0 = 1$, $a_1 = 2$
$a_n = a_{n-1} + 3a_{n-2}$
$a_0 = 1$
$\boxed{a_1 = 2}$
$a_2 = a_{2-1} + 3a_{2-2}$
$= a_1 + 3$
$= 2 + 3 \times 1 = \boxed{a_2 = 5}$

$a_3 = a_{3-1} + 3a_{3-2}$
$= a_2 + 3a_1$
$= 5 + 3\times 2$
$= 5+6$
$\boxed{a_3 = 11}$

$a_4 = a_{4-1} + 3a_{4-2}$
$= a_3 + 3a_2$
$= 11 + 3\times 5$
$= 11+15$
$\boxed{a_4 = 26}$

Put $n=2 \Rightarrow a_2 = 2^2 + (5)(3^2)$
$= 4 + 5\times 9$
$= 4+45 \Rightarrow \boxed{a_2 = 49}$

b) Given $a_n = 2^n + (5)3^n$

Put $n=3 \Rightarrow a_3 = 2^3 + 5\times 3^3$
$= 8 + 5\times 25$
$\boxed{a_3 = 143}$

To prove that $a_4 = 5a_3 - 6a_2$

Now take L.H.S

put $n=4 \Rightarrow a_4 = 2^4 + 5 3^4$
$= 16 + 5\times 81$
$\boxed{a_4 = 421} \rightarrow (1)$

Now take R.H.S

$a_4 = 5a_3 - 6a_2$
$= 5\times 143 - 6\times 49$
$= 415 - 294$
$\boxed{a_4 = 421} \rightarrow (2)$

$(1) \Rightarrow (2)$

$421 = 421$

LHS = RHS

Hence the result.

c) $a_n = a_{n-1} + a_{n-3}$, $a_0 = 1$, $a_1 = 2$, $a_2 = 0$

$a_0 = 1$, $a_1 = 2$, $a_2 = 0$

$a_n = a_{n-1} + a_{n-3}$

Put $n=3$ $a_3 = a_{3-1} + a_{3-3}$
$= a_2 + a_0$
$= 0 + 1$
$= \boxed{a_3 = 1}$

$a_4 = a_{4-1} + a_{4-3}$
$= a_3 + a_1$
$= 1+2 \Rightarrow \boxed{a_4 = 3}$

Hence the result.

4) let $a_n = 2^n + (5)(3^n)$ for $n = 0, 1, 2, 3 \ldots$

a) Find a_0, a_1 and a_2

b) Show that $a_4 = 5a_3 - 6a_2$

a) Given $a_n = 2^n + (5)(3^n)$

put $n=0 \Rightarrow a_0 = 2^0 + 5(3^0)$

$= 1 + 5 \times 1 \Rightarrow 1+5 = \boxed{a_0 = 6}$

Put $n=1 \Rightarrow a_1 = 2^1 + 5(3^1)$

$= 2 + 5 \times 3$

$= 2 + 15 = \boxed{a_1 = 17}$

Put $n = 2 \Rightarrow a_2 = 2^2 + (5)(3^2)$

$= 4 + 5 \times 9 \Rightarrow 4 + 45$

$\boxed{a_2 = 49}$

b) Given $a_n = 2^n + (5)^{3n}$

Put $n = 3 \Rightarrow a_3 = 2^3 + 5 \times 3^3$

$= 3 + 5 \times 27 = \boxed{a_3 = 143}$

To prove that $a_4 = 5a_3 - 6a_2$

Now take L.H.S

Put $n=4 \Rightarrow a_4 = 2^4 + 5\,3^4$

$= 16 + 5 \times 81$

$\boxed{a_4 = 421}$

Now take R.H.S

$a_4 = 5a_3 - 6a_2$

$= 5 \times 143 - 6 \times 49$

$= 715 - 294$

$a_4 = 421$

① ⇒ ②

$421 = 421$

LHS = RHS.

5) Show that the sequence $\{a_n\}$ is a solution of the recurrence relation.

$a_n = a_{n-1} + 2a_{n-2} + 2n - 9$ if - ①

a) $a_n = 3(-1)^n + 2^n - n + 2$

b) $a_n = 7 \cdot 2^n - n + 2$

Given

a) $a_n = 3(-1)^n + 2^n - n + 2$

put $n = n-1 \Rightarrow a_{n-1} = 3(-1)^{n-1} + 2^{n-1} - (n-1) + 2$

$= 3(-1)^n (-1)^{-1} + 2^n 2^{-1} - n + 2$

$= 3(-1)^n (-1) + 2^n (1/2) - n + 2$

$a_{n-1} = -3(-1)^n + 1/2\, 2^n - n + 2 \rightarrow$ equ ①

put $n = n-2 \Rightarrow a_{n-2} = 3(-1)^{n-2} + 2^{(n-2)} - (n-2) + 2$

$= 3(-1)^n (-1)^{-2} + 2^n 2^{-2} - n + 2 + 2$

$= 3(-1)^n + 2^n (\frac{1}{4}) - n + 2$

$a_{n-2} = 3(-1)^n + \frac{1}{4} 2^n - n + 4$

$2a_{n-2} = 3(-1)^n + \frac{1}{4} 2^n - n + 4$

$2a_{n-2} = 2(3(-1)^n + \frac{1}{4} 2^n - n + 4)$

$= 6(-1)^n + \frac{2}{4} 2^n - 2n + 8$

$2a_{n-2} = 6(-1)^n + \frac{1}{2} 2^n - 2n + 8 \rightarrow$ ②

equ ① and ② substating in equ (*)

$a_{n-1} + 2a_{n-2} + 2n - 9 =$

$= -3(-1)^n + \frac{1}{2} 2^n - n + 3 + 6(-1)^n$

$+ \frac{1}{2} 2^n - 2n + 8 + 2n - 9$

$a_{n-1} + 2a_{n-2} + 2n - 9 = \frac{7}{2} 2^n - n + 3 + \frac{7}{2} 2^n -$

$2^n + 8 + 2n - 9$

$= 7 2^n - n + 2$

$a_{n-1} + 2a_{n-2} + 2n - 9 = a_n.$

⑥ let $a_n = 2^n + 5(3^n)$

for $n = 0, 1, 2, 3 \ldots$

find $a_0, a_1, a_2, a_3, a_4, a_5$

sol: Given:

$a_n = 2^n + 5(3^n)$

Put $n = 0$

$a_0 = 2^0 + 5(3^0)$

$= 1 + 5(1)$

$= 1 + 5 = 6$

$\boxed{a_0 = 6}$

put $n = 1$

$a_1 = 2^1 + 5(3^1)$

$= 2 + 5 \times 3$

$= 2 + 15 = 17$

$\boxed{a_1 = 17}$

Put $n = 2$

$a_2 = 2^2 + 5(3^2)$

$= 4 + 5 \times 9 = 4 \times 45 = 49$

$\boxed{a_3 = 49}$

put $n = 3$

$a_3 = 2^3 + 5(3^3)$

$= 8 + 5 \times 3 \times 3 \times 3$

$= 8 + 5 \times 27$

$= 8 + 135$ $\boxed{a_3 = 143}$

$a_4 = 2^4 + 5\times3^4$

$= 16+5\times3\times3\times3\times3$

$= 16+5\times9\times9$

$= 16+5\times81$

$= 16+405$

$\boxed{a_4 = 421}$

$a_5 = 2^5 + 5\times5^5$

$= 32+5\times3\times3\times3\times3\times3$

$= 32+5\times9\times9\times3$

$= 32+5\times81\times3$

$a_5 = 32+1215$

$\boxed{a_5 = 1247}$

Defn:

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