Matrix operations

Addition of two matrix

we defined the addition of two matrix A and B only when the matrices A and B have the same order If $A = [a_{ij}]$ and $B = [b_{ij}]$ are $m \times n$ then the $m \times n$ matrix

$$C = [c_{ij}] \text{ where}$$
$$c_{ij} = a_{ij} + b_{ij}$$
$$1 \le i \le m$$
$$1 \le j \le n$$

is said to be the sum (or) addition of A & B and we write $C = A+B$

$A = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 0 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 3 & -5 \\ 2 & 0 & -10 \end{pmatrix}$

Then:

$$C = A+B = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 0 & 7 \end{pmatrix} + \begin{pmatrix} 0 & 3 & -5 \\ 2 & 0 & -10 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 2+3 & 3-5 \\ 5+2 & 0+0 & 7-10 \end{pmatrix} = C = \begin{pmatrix} 1 & 5 & -2 \\ 7 & 0 & -3 \end{pmatrix}$$

$1 \le i \le m$ and $1 \le j \le n$ is called the Scalar Multiplication of A by K

and wc_2 and $B = KA$

Ex :

If $A \begin{pmatrix} 1 & 2 & 3 \\ 5 & 0 & 7 \end{pmatrix}$ then

$$5A = 5 \begin{pmatrix} 1 & 2 & 3 \\ 5 & 0 & 7 \end{pmatrix}$$

$$5A = \begin{bmatrix} 5 & 10 & 15 \\ 25 & 0 & 35 \end{bmatrix}$$

If $k = -1$ then kA is denoted by $-A$

Multiplication of two matrices:

we defined product AB of the matrices $A = [a_{ij}]_{m \times n}$

$B = [b_{ij}]_{r \times 5}$ only when $n = r$ ie) A

The number colomns of A = the number of rows are B.

we say that $A = [a_{ij}]_{m \times n}$ $B = [b_{ij}]_{r \times s}$ are compoducable if $n = r$ if + definition $A = [a_{ij}]$, $B = [b_{ij}]$ are $m \times n$ and $n \times r$ matrices

$$C = [c_{ij}]_{m \times n}$$

$$c_{ij} = a_{ij} + a_{i2} + a_{j2} + \dots + a_{in} b_{nj} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

for $i = 1, 2 \dots m$ and $j = 1, 2, \dots r$

Ex: If $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{pmatrix}$

sol:

$$AB = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \times 1 + 1 \times (-1) + 2 \times 2 \\ 1 \times 1 \times 2 \times (-1) + 3 \times 2 \\ 2 \times 1 + 3 \times (-1) + 4 \times 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & +4 \\ 1 & -2 & +6 \\ 2 & -3 & +8 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \times (-2) + 1 \times 0 + 2 \times (-1) \\ 1 \times (-2) + 2 \times 0 + 3 \times (-1) \\ 2 \times (-2) + 3 \times 0 + 4 \times (-1) \end{pmatrix}$$

$$\begin{pmatrix} 0 + 0 - 2 \\ -2 + 0 - 3 \\ -4 + 0 - 4 \end{pmatrix} \begin{pmatrix} -2 \\ -5 \\ -1 \end{pmatrix}$$

Hence

$$AB = \begin{pmatrix} 3 & -2 \\ 5 & -5 \\ 7 & -8 \end{pmatrix}$$

If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{pmatrix}$

To verify that $AB = BA$

Given $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0-6 & -2+0+2 & 3+0+4 \\ 0+2-6 & 0+3+2 & 0+-1+4 \\ 1+4+0 & -2+6+0 & 3-2+0 \end{pmatrix}$$

$$AB = \begin{pmatrix} 5 & 0 & 7 \\ -4 & 5 & 3 \\ 5 & 4 & 1 \end{pmatrix} \rightarrow (1)$$

$$BA = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1-0+3 & 0+2+6 & 2-4+0 \\ 2+0-1 & 0+3-2 & 4+6-0 \\ -3+0+2 & 0+1+4 & -6+2+0 \end{pmatrix}$$

$$BA = \begin{pmatrix} 4 & 4 & -2 \\ 1 & 1 & 10 \\ -1 & 5 & -4 \end{pmatrix} \rightarrow (2)$$

Hence $(1) \neq (2)$

$AB \neq BA$

Hence the result.