

QUESTIONS

4.7.25

1) What are logical connectivities? Construct the truth table.

Proof:- Logical connectivities are symbols / words used in logic to connect two / more proposition (statement) to form a more complex logical expression.

These connectivities help us determine the truth value of compound statement based on the truth values of the individual statement.

Common logical connectivities:-

Name	Symbol	Meaning
AND	$\wedge$	True if both
OR	$\vee$	True if atleast one true
NOT	$\sim$	Negation (opposite)
IMPLIES	$\longrightarrow$	If then
BICONDITIONAL	$\longleftrightarrow$	If and only if (iff)

Truth table for logical connectives:-

Let P and Q be two statements.

P	Q	$\sim P$	$\sim Q$	$P \wedge Q$	$P \vee Q$	$P \longrightarrow Q$	$P \longleftrightarrow Q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	F	F	T	T	F
F	F	T	T	F	F	T	T

Explanation (in simple terms):-

- i) Negation of P ($\neg P$): opposite of P
- ii) $\neg Q$: Opposite of Q
- iii) $P \wedge Q$: True only when both P & Q are true.
- iv) $P \vee Q$: True if atleast one of P/Q is true.

- v) $P \rightarrow Q$: False ^{only} when P is true Q is false.
- vi) $P \leftrightarrow Q$: True, when both P & Q are same.
(Both are either true or false)

2. $P \wedge Q \rightarrow P$ is a tautology using the table of propositional equivalence.

Soln:-

It is enough to show that $P \wedge Q \rightarrow P$ is a tautology.

P	Q	$P \wedge Q$	$P \wedge Q \rightarrow P$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$\therefore P \wedge Q \rightarrow P$ is a tautology.

3) use the truth table method to determine whether $(\sim(P \vee Q) \wedge (Q \rightarrow \sim R \wedge \sim P) \wedge (P \vee R))$ denoted by ϕ is satisfiable.

4) Proof: List all combinations of truth valuable of P, Q, R. There are three variables:

$$2^n \rightarrow 2^3 \rightarrow 8$$

P	Q	r	$\sim P$	$(\sim P \vee Q)$	$\sim r$	$\sim r \wedge \sim P$	$Q \rightarrow \sim r \wedge \sim P$	$P \vee r$	ϕ
T	T	T	F	T	F	F	F	T	F
T	T	F	F	T	T	F	F	T	F
T	F	T	F	F	F	F	T	T	F
T	F	F	F	F	T	F	T	T	F
F	T	T	T	T	F	F	F	T	F
F	T	F	T	T	T	T	T	F	F
F	F	T	T	T	F	F	T	T	T
F	F	F	T	T	T	T	T	F	F

Conclusion:-

In all 8 cases, ϕ is false. So, ϕ is not satisfiable.

Hence, the result.

4) What are the rules of inference?

Sol:-

Basic rules of inference:

1. Negation (NOT: $\sim$, $\neg$)

Opposite meaning of the statement.

Eg:- It will rain today: P

It will not rain today: $\neg P$

Truth table:-

P	$\sim P$
T	F
F	T

2. Conjunction (AND: $\wedge$)

If both P and Q are true, the $P \wedge Q$ is true.

Otherwise false. Eg:- Prema will go for shopping: P

Prema will go for temple: Q

$P \wedge Q$: Prema will go for shopping and temple.

Truth table :-

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

3) Disjunction (OR: $\vee$)

If either P or Q is true, then $P \vee Q$ is true; Otherwise false.

Eg:- Kavya will go for a walk: P
Kavya will read a book: Q

$P \vee Q$: Kavya will go for a walk or read a book.

4) Conditional (if, then: $\rightarrow$)

If P is true & Q is false, then $P \rightarrow Q$

Truth Table:

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional

4) Conjunction (if, then: $\rightarrow$)

If P is true & Q is false, then $P \rightarrow Q$ is true; Otherwise false.

Eg:- You are a student: P

You have a student ID: Q

$P \rightarrow Q$: If you are a student, then you have a student ID.

Truth Table:

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

- 5) Bi-conditional (if and only if : $\leftrightarrow$) [iff]
If P and Q are not equal, then it is false. Otherwise it is true.

Eg:- A number is even - P
It is divisible by 2 - Q

$P \leftrightarrow Q$: A number is even ~~iff~~ it is divisible by 2.

Truth Table:

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T