Logarathmic Functions:

$\Rightarrow$ Logarithmic function is an ~~inver~~ inverse function of an exponential function.

$\Rightarrow$ for example. if y is equal to the value of the logarithm of constant base and variable power or index or exponent,

$\Rightarrow$ we obtain the "Logarithmic function".

$\Rightarrow$ It is of the form

$$y = \log a^x \text{ or } y = \log e^x$$

for $x > 0$, $a > 0$ and $a \neq 1$

$$y = \log a^x \text{ if and only if } x = a^y$$

$\Rightarrow$ The base of the logarithm is a.

$\Rightarrow$ This can be read it as log base a of x.

$\Rightarrow$ The most 2 common bases used in logarithmic functions are base 10 and base e.

Common Logarithmic Function:-

⇒ The logarithmic function with base 10 is called the common logarithmic function and it is denoted by $\log_{10}$ or simply log.

$$f(x) = \log_{10} x$$

Natural Logarithmic Function:-

⇒ The logarithmic function to the base e is called the natural logarithmic function and it is denoted by $\log_e$

$$f(x) = \log_e x$$

Logarithmic functions Properties:-

⇒ Logarithmic function have some of the properties

$\boxed{\text{Logarithmic}}$

Basic contition

Exponential form → Logarithmic

$a^x = N \rightarrow \log_a N = x$

$3^2 = 9 \Rightarrow \log_3 9 = 2$

⇒ log newer comes in Negative.

⇒ log '0' Does not exist

(i) $N > 0$ (ii) $a > 0$, $a \neq 1$

$\log_a 1 = 0 \Rightarrow a^0 = 1$ any thing for 0 is ⇒ 1

$\log_N N = 1 \Rightarrow N^1 = N$

$\log_{1/N} N = \log_N 1/N = -1$ $\boxed{\text{Resiprogal}}$

$a^{\log_a N} = N$ Eg: $2^{\log_2 7} = 7$

Properties:-

(1) Product Rule

$$\log_b MN = \log_b M + \log_b N$$

⇒ Multiply two numbers with same base, then add the exponents

Ex: $\log 30 + \log 2 = \log 60$

(2.) Quotient Rule

$$\log_b M/N = \log_b M - \log_b N$$

⇒ Divide two numbers with same base, Subtract the exponents.

Ex: $\log_8 56 - \log_8 7 = \log_8 (56/7) = \log_8 8 = 1$

(3) Power Rule

⇒ Raise an exponential expression to Power and multiply the exponents.

$$\log_b M^P = P \log_b M$$

Ex: $\log 100^3 = 3 . \log 100 = 3 \times 2 = 6$

(4) Zero Exponent Rule

$$\log a^1 = 0$$

Change of Base Rule

$$\log_b(x) = \ln x / \ln b \text{ or } \log_b(x) = \log_{10} x / \log_{10} b$$

$$\Downarrow$$

$$\frac{\log_a \cancel{x}}{\log_a b}$$

Other Important rule:

Base Power Formula

$\Rightarrow \log_{a^k} m = \frac{1}{k} \log_a m$

$\boxed{\log_{a^k} b^c = \frac{c}{k} \log_a b}$ extra

Note: Common | natural

$\log_{10}$ | $\log_e$

$[e = 2.718]$ app

not necessary

$\log_e a = 2.303 \times \log_{10} a$

or

$\frac{\log a}{\log e}$

$\log_{10} a = 0.434 \log_e a$

Example 1 :-

Use the Properties of logarithms to write as a Single logarithm for the given equation:

$5 \log_q x + 7 \log_q y - 3 \log_q z$

By using the Power rule,

$\log_b m^p = p \log_b m$, we can write the given equation

$5 \log_q x + 7 \log_q y - 3 \log_q z =$

$\boxed{\log_q x^5 + \log_q y^7 - \log_q z^3}$

From Product rule, $\log_b MN = \log_b M + \log_b N$

$$5\log_a x + 7\log_a y - 3\log_a z =$$

$$\log_a x^5 y^7 - \log_a z^3$$

From Quotient rule, $\log_b M/N = \log_b M - \log_b N$

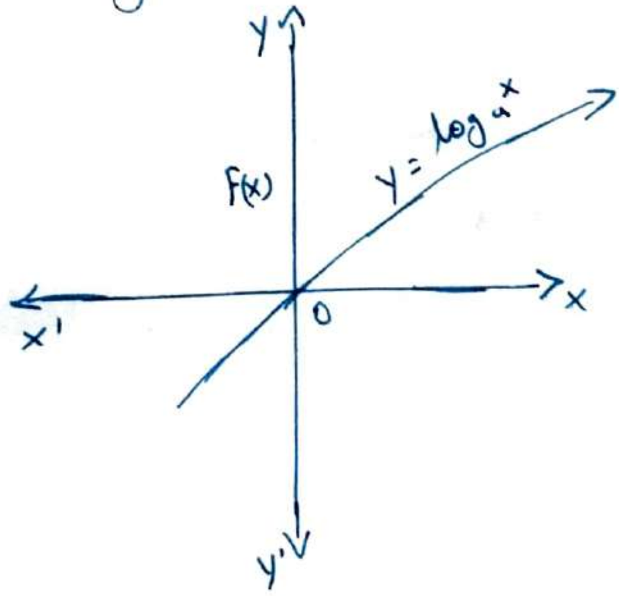
$$5\log_a x + 7\log_a y - 3\log_a z = \log_a (x^5 y^7 / z^3)$$

Graphical Representation

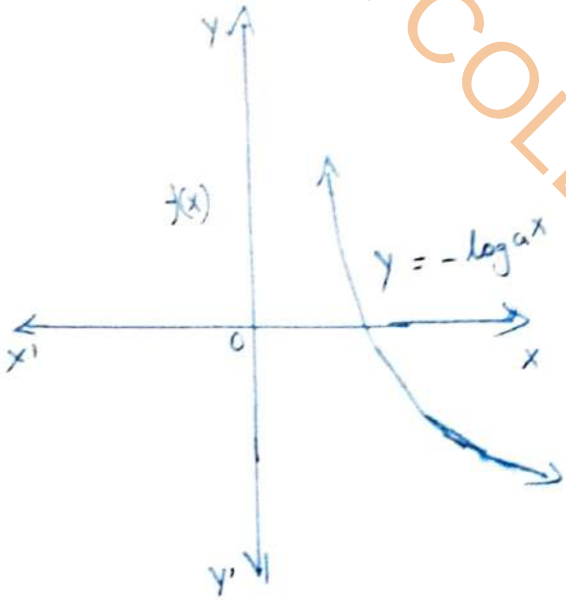
⇒ The graph of the logarithmic function is shown

⇒ there is a family of curves depending upon the value of the base 'a'

i) a is greater than 1



(ii) a is less than 1.



Ex: Graph the logarithmic function

$F(x) = 2\log_3(x+1)$

Solution:-

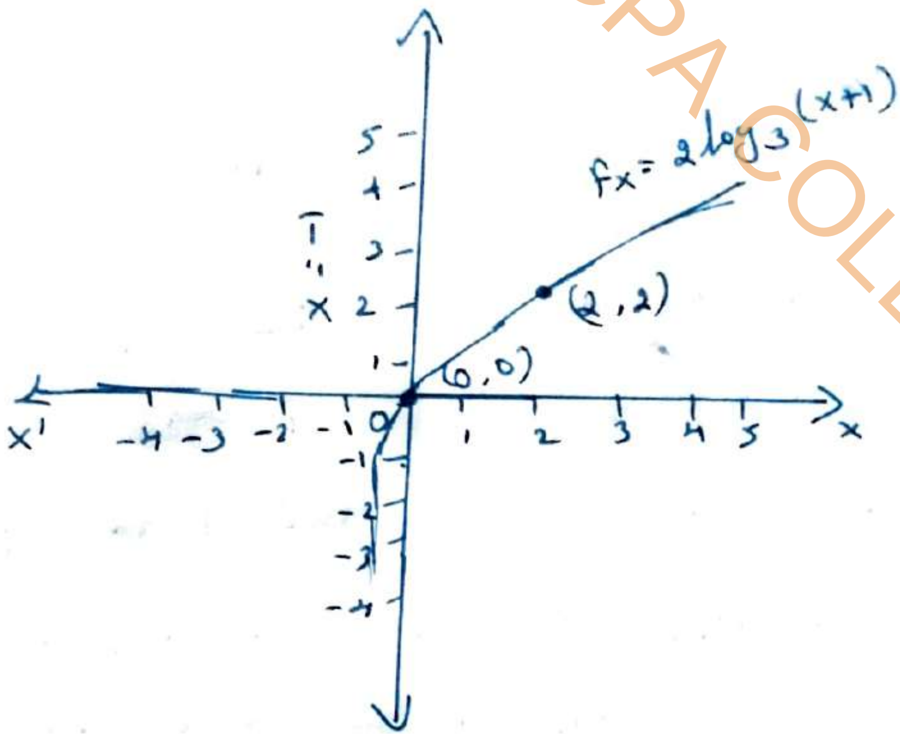
Here, the base is $3 > 1$. So the curve would be increasing

For domain: $x + 1 > 0 \Rightarrow x > -1$ So domain $= (-1, \infty)$

vertical asymptote is $x = -1$

- At $x = 0$, $y = 2\log_3(0+1) = 2\log_3 1 = 2(0) = 0$
- At $x = 2$, $y = 2\log_3(2+1) = 2\log_3 3 = 2(1) = 2$

Thus $(0, 0)$ and $(2, 2)$



fx = 2log3(x+1)
(2,2)
(0,0)
X
X'
x
5
4
3
2
1
-1
-2
-3
-4