Inverse of a square matrix

The A of order n is said to be invertible if their exist a square B of order n such that AB = BA = In and B is called the inverse of a square matrix.

and is denoted by A^{-1}

(ie) $A^{-1} = \frac{1}{|A|} \text{adj} A$

1) Find the inverse of matrix $\begin{pmatrix} 2 & 4 & -1 \\ 0 & 3 & 7 \\ 8 & 1 & 8 \end{pmatrix}$

10m

$$\boxed{A^{-1} = \frac{1}{|A|} \text{adj} A} \rightarrow (1)$$

Given $A = \begin{pmatrix} 2 & 4 & -1 \\ 0 & 3 & 7 \\ 8 & 1 & 5 \end{pmatrix}$

$$|A| = 2(15-7) = 4(0-56) - 1(0-24)$$
$$= 2(8) + 224 + 24$$
$$= 16 + 224 + 24$$
$$|A| = 264 = 264 \rightarrow (2)$$

we have already abtain the adjaint of A (adj A) in problem-1

$$\text{adj} A = \begin{pmatrix} 8 & -21 & 31 \\ 56 & 18 & -14 \\ -24 & 30 & 6 \end{pmatrix} - (3)$$

we get

$$A^{-1} = \frac{1}{264} \begin{pmatrix} 8 & -21 & 31 \\ 56 & 18 & -14 \\ -24 & 30 & 6 \end{pmatrix}$$

2) 10m Find the inverse of A

(12)

$$A = \begin{pmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ -10 & -1 & -4 \end{pmatrix}$$

Sol :

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A \rightarrow (1)$$

Given $A \begin{pmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ -10 & -1 & -4 \end{pmatrix}$

$$|A| = \begin{vmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ -10 & -1 & -4 \end{vmatrix}$$

$= 8(-4+2) + 1(-20+20) - 3(5-10)$

$= 8(-2) + 0 + -3(-5)$

$= -16 + 15 = -1$

$|A| = -1 \rightarrow (2)$

(adj A) is problem no 2

$$\operatorname{adj} A = \begin{pmatrix} -2 & -1 & 1 \\ 0 & -2 & -1 \\ -5 & 2 & 3 \end{pmatrix} \rightarrow (3)$$

equ (2) & equ (3) sub in equ (1)

we get

$$A^{-1} = \frac{1}{(-1)} \begin{pmatrix} -2 & -1 & 1 \\ 0 & -2 & -1 \\ -5 & 2 & 3 \end{pmatrix} \quad A^{-1} = \begin{pmatrix} -2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & -2 & -3 \end{pmatrix}$$

---

3) Find the inverse of the matrix $\begin{pmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A \rightarrow (1)$$

Given $A = \begin{pmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$

$$|A| = \begin{vmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}$$

$= 3(-3-4) + 43(2+0) + 4(-2+0)$

$= 3(-7) + 3(2) + 4(0)$

$= -21 + 6 + 4$

$= -15 + 4$

$|A| = -11 \rightarrow (2)$

we have already obtain (adj A) in problem no: 3

$$\operatorname{adj} A = \begin{pmatrix} -1 & 7 & -24 \\ 2 & -3 & 4 \\ 2 & -3 & 15 \end{pmatrix} \rightarrow (3)$$

(3) & equ (2) sub in equ (1) we get

$$A^{-1} = \frac{1}{-11} \begin{pmatrix} -1 & 7 & -24 \\ 2 & -3 & 4 \\ 2 & -3 & 15 \end{pmatrix}$$