Graph Terminology and special Types of Graphs:

undirected Graph:

Adjacent:

Two vertices u and v in an undirected graph G are called adjacent (or neighbours) in G if u and v are endpoints of an edge e of G. such an edge e is called incident with the vertices u and v and e is said to connect u and v.

Neighborhood:

The set of all neighbours of a vertex v of $G = (V,E)$, denoted by $N(v)$, is called the neighborhood of v. If A is a subset of V, we denote by $N(A)$ the set of all vertices in G that are adjacent to at least one vertex in A. So, $N(A) = \bigcup N(v)$.

Degree of a vertex:

The degree of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex. The degree of the vertex v is denoted by deg(v).

A vertex of degree zero is called isolated.

A vertex is a pendant if and only if it has a degree one.

Handshaking Theorem:

Let $G = (V, E)$ be an undirected graph with m edges. Then $2m = \sum_{v \in V} \deg(v)$.

Directed Graph:

Adjacency:

When (u,v) is an edge of the in-degree of a vertex, v, denote the graph G with directed edges, u is said to be adjacent to v and v is said to be adjacent from u. The vertex u is called the initial vertex of (u,v), and v is called the terminal or end vertex of (u,v). The initial vertex and terminal vertex of a loop are the same.

Degree of a vertex:

In a graph with directed edges the indegree of a vertex v, denoted by $deg^-(v)$, is the number of edges with v as their terminal vertex. The out-degree of v, denoted by $deg^+(v)$, is the number of edges with v as their initial vertex.

Handshaking Theorem for Directed Graph (Theorem 3)

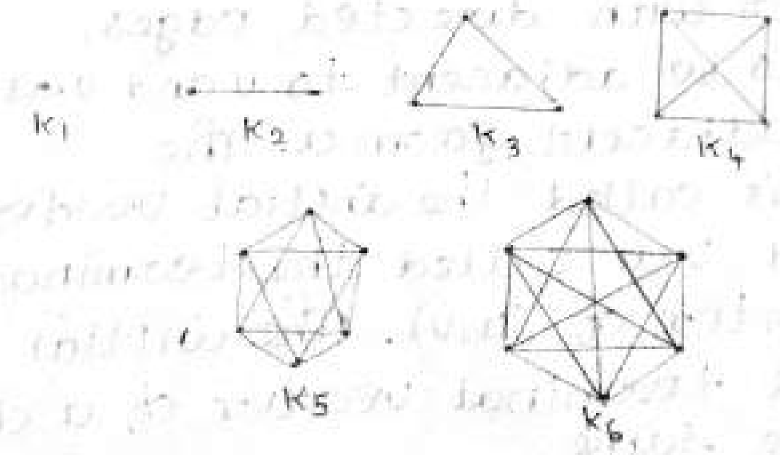
Let $G = (V,E)$ be a graph with directed edges. Then $\sum_{v \in V} deg^-(v) = \sum_{v \in V} deg^+(v) = |E|$

Special Graphs:

Complete Graphs:

A Complete graph on n vertices, denoted by K_n, is a simple graph that contains exactly one edge between each pair of distinct vertices.

Has $\frac{n(n-1)}{2}$ edges.



Cycles :

A cycle C_n, $n \geq 3$, consists of n vertices $v_1, v_2, \dots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\} \dots \{v_{n-1}, v_n\}$ and $\{v_n, v_1\}$. Has n edges.

Representing Graphs and Graphs :