

Generalized pigeonhole principles

Defn:

If n pigeonholes are occupied
 $kn+1$ more pigeons, where k is a
occupied positive integer then at least
one pigeonhole is occupied by $k+1$
(or) more pigeons

Example:

find the minimum number of
students in a class to be sure
that three (3) of them are born
in the same month.

sol:

here: $n = 12$ months (or) pigeonholes
and $k+1 = 3$

$$k = 3 - 1 \Rightarrow k = 2$$

$$k = 2, n = 12$$

Hence among any $kn+1 = 2 \times 12 + 1$
 $= 24 + 1$
 $= 25$

$$\boxed{kn+1 = 25}$$

Hence students (pigeon) three of

them of born in the same month.

Red

Problem,

- 1) 7 members of a family have total Rs. 2886 in their pockets show that atleast one of them must have atleast Rs 416 in his pockets

Solu:

Let us assume that members \rightarrow Pigeon holes

Rupees \rightarrow pigeons $\rightarrow k = 2886$ $n = 7$

Now, 2886 pigeons are to be assigned to 7 pigeonhole using the extant Pigeon hole principle.

$$\text{i.e.) } \frac{k-1}{n} + 1$$

Here $k = 2886, n = 7$

$$\Rightarrow \frac{k-1}{n} + 1 \Rightarrow \frac{2886-1}{7} + 1 \Rightarrow \frac{2885}{7} + 1$$
$$\Rightarrow 415 + 1 \Rightarrow 416$$

Hence, there are Rs 416 in one members pockets.

- 2) If 9 Books are to be kept in 4 shelves, there must be atleast one shelf which contain atleast 3 Books.

Sol:

Let us assume that Books \rightarrow pigeon $\rightarrow k = 9$
Shelves \rightarrow pigeon holes $\rightarrow n = 4$

Now, 9 pigeons are to be assign to 4 pigeon hole using the extant pigeon hole principle.

Then $\frac{k-1}{n} + 1$, here $k=9, n=4$

$$\Rightarrow \frac{9-1}{4} + 1 \Rightarrow \frac{8}{4} + 1 \Rightarrow 2 + 1 \Rightarrow 3 \text{ Books}$$

Hence, there are 3 books in atleast one shelf, Hence the result.

2) 8m

How many people must you have to guarantee atleast 9 of the birthdays same day of the week.

Solu.

Let us assume that the days week \rightarrow pigeonholes $\rightarrow n=7$

people \rightarrow pigeons $\rightarrow k=?$

Now, 7 pigeonholes and we have to find pigeons using the extent pigeon hole principle.

$$\text{i.e.) } \frac{k-1}{n} + 1 = k = ? \quad n=7$$

$$\frac{k-1}{7} + 1 = 9$$

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$$\frac{k-1+7}{7} = 9 \Rightarrow \frac{k+6}{7} = 9 \Rightarrow k+6 = 9 \times 7$$

$$k+6 = 63 \Rightarrow k = 63 - 6 \Rightarrow \boxed{k=57}$$

Thus, There must be 57 people guarantee that atleast 9 of them will have birth days in same day of the week,

Hence the Result.

4) Show that if 30 dictionary in a library in contain a total of 61327 pages. then one of the dictionary must have 2045 pages

Solut:

Let us assume that

pages \rightarrow pigeons $\rightarrow k=61327$

dictionaries \rightarrow pigeonholes $\rightarrow n = 30$

assign to each page in the dictionary in which appear then by extent pigeon hole principle.

Thus, $\frac{k-1}{n} + 1$ Here, $n = 30$
 $k = 61327$

$$\frac{61327-1}{30} + 1 \Rightarrow \frac{61326}{30} + 1 \Rightarrow 2044 + 1$$

2045 pages.

Hence, one dictionary contains 2045 pages, \therefore Hence the Result.