Find the eigen values and eigen vectors

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

Ans :

Given $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$

$\therefore$ The char equ $= |A - \lambda I| = 0 \rightarrow (1)$

$$|A - \lambda I| = \begin{vmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{vmatrix} - \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)\left[(3-\lambda)(2-\lambda) - 2\right]$$

$$- 2\left[1(2-\lambda) - 1\right] + 1\left(2 - (3-\lambda)\right)$$

$$= (2-\lambda)\left[6 - 3\lambda - 2\lambda + \lambda^2 - 2\right]$$

$$- 2\left[2 - \lambda - 1\right] + \left[2 - 3 + \lambda\right]$$

$$= (2-\lambda)(\lambda^2 - 5\lambda + 4) - 2(1-\lambda) + (-1+\lambda)$$

$= 2\lambda^2 - 10\lambda + 8 - \lambda^3 + 6\lambda^2 - 4\lambda - 2 + 2\lambda - 1 + \lambda$

$= -\lambda^3 - 7\lambda^2 - 11\lambda + 8 - 3$

$\Rightarrow -\lambda^3 + 7\lambda^2 - 11\lambda + 5 = 0$

1	-1	$+7$	-11	5
	0	-1	6	-5
1	-1	6	-5	$\boxed{0}$
	0	-1	5	
5	-1	5	$\boxed{0}$	
	0	-5		
	$\boxed{-1}$	$\boxed{0}$		

$\boxed{\lambda = 1, 1, 5}$

$\therefore$ The eigen values are $\lambda = 1, 1, 5$

$$(A - \lambda I) = \begin{pmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2 \end{pmatrix} \rightarrow (*)$$

Now take $\lambda = 1$ in sublituting equ number $(*)$

$$= \begin{pmatrix} 2-1 & 2 & 1 \\ 1 & 3-1 & 1 \\ 1 & 2 & 2-1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{pmatrix} = 0$$

$= x_1 + 2x_2 + x_3 = 0 \rightarrow (2)$

$= x_1 + 2x_2 + x_3 = 0 \rightarrow (3)$

$= x_1 + 2x_2 + x_3 = 0 \rightarrow (4)$

Now take (2) & (3) by using cross multiplying method

Since cy costant x y

$$\begin{pmatrix} 2 & 1 & 1 & 2 \\ 2 & 1 & 1 & 2 \end{pmatrix}$$

$$\frac{x_1}{2-2} = \frac{x_2}{0} = \frac{x_3}{0}$$

The eigen vectors = (0,0,0)

Now take $\lambda = 5$

$$= \begin{pmatrix} 2-5 & 2 & 1 \\ 1 & 3-5 & 1 \\ 1 & 2 & 2-5 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{pmatrix} = 0$$

$= -3x_1 + 2x_2 + 1x_3 = 0 \rightarrow (2)$

$= x_1 + -2x_2 + 1x = 0 \rightarrow (3)$

$= x_1 + 2x_2 - 3x_3 = 0 \rightarrow (4)$

Now take (2) & (3) by using Cross multiplying method

Since y Costant x y

$$\begin{pmatrix} 2 & 1 & -3 & 2 \\ -2 & 1 & 1 & 2 \end{pmatrix}$$

$$\frac{x_1}{2+2} = \frac{x_2}{1+3} = \frac{x_3}{+6-2}$$

$$\frac{x_1}{4} = \frac{x_2}{4} = \frac{x_3}{4}$$

The eigen vectors = $(4,4,4) \Rightarrow (1,1,1)$

Find the eigenvalue and eigen vectors

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & 1 \\ 2 & -1 & 2 \end{bmatrix}$$

proof :

Char equ A is $|A - \lambda I| = 0$

$$A - \lambda I = \begin{vmatrix} 6 & -2 & 2 \\ -2 & 3 & 1 \\ 2 & -1 & 2 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$A - \lambda I = \begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & 1 \\ 2 & -1 & 2-\lambda \end{vmatrix}$$

$= 6-\lambda\,[(3-\lambda)(2-\lambda)-1]$
$+2\,[-2(2-\lambda)+2]$
$+2\,[2(2(3-\lambda)\,(6-\lambda)\,(9-3\lambda-3\lambda+\lambda^2)+2[-6-2\lambda+2]$
$2\,[2(6-2\lambda)]$

$(6-\lambda)\,[x^2-6\lambda+9-1]+2(2\lambda-4)+2[2\lambda-4]=0$

$(6-\lambda)\,[x^2 6\lambda+9-1]+4\lambda+4\lambda-8=0$

$=(6-\lambda)(\lambda^2-6\lambda)+8)+4\lambda+4\lambda-8=0$

$=(6-\lambda)\,[\lambda^2-6\lambda+8)+4\lambda-8+4\lambda-8=0$

$=6\lambda^2-36\lambda+48-\lambda^3+6\lambda^3-8\lambda+4\times 8\;4\lambda-8=0$

$=-\lambda^3+12\lambda^2-36\lambda+32=0$

$(\lambda-2)\,(-x^2+10\lambda-16)=0$

	-1	12	-36	32
2	0	-2	20	32
	-1	10	-16	$\boxed{0}$
2	0	-2	-16	
	-1	8	$\boxed{0}$	
8	0	-8		
	$\boxed{-1}$	$\boxed{0}$		

$\boxed{\lambda = 2,2,8}$

$\therefore$ The If $\lambda=2$ $\begin{pmatrix} 6-2 & -2 & 2 \\ -2 & 3-2 & -1 \\ 2 & -1 & 3-2 \end{pmatrix}$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} = 0$$

$= 4x_1 - 2x_2 + 2x_3 = 0 \rightarrow (2)$

$= -2x_1 + x_2 - x_3 = 0 \rightarrow (3)$

$= 2x_1 - x_2 + x_3 \rightarrow (4)$

Now take (2) and (3) by using cross multiplying method.

(since y constant α y)

$\begin{pmatrix} -2 & 2 & 4 & 2 \\ 1 & -1 & -2 & 1 \end{pmatrix}$

$$\frac{x_1}{2-2} = \frac{x_2}{-4+4} = \frac{x_3}{4-4}$$

$$= \frac{x_1}{0} = \frac{x_2}{0} = \frac{x_3}{0}$$

The eigen vector = (0, 0, 0)

If $\lambda = 8$ $\begin{vmatrix} 6-8 & -2 & 2 \\ -2 & 3-8 & -1 \\ 2 & -1 & 3-8 \end{vmatrix}$ 1)

$$= \begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix}$$

$= -2x_1 - 2x_2 + 2x_3 \rightarrow (2)$

$= -2x_1 - 5x_2 - x_3 \rightarrow (3)$

$= 2x_1 - x_2 - 5x_3 \rightarrow (4)$

Now take (2) and (3) using cross multiplying method

(y constant x y)

$$= \begin{pmatrix} -2 & 2 & -2 & -2 \\ -5 & -1 & -2 & -5 \end{pmatrix}$$

$$= \frac{x_1}{+2+10} = \frac{x_2}{-4-2} = \frac{x_3}{+10-4}$$

$$= \frac{x_1}{12} = \frac{x_2}{-6} = \frac{x_3}{+6}$$ 2)

The eigen vectors is (2, -1, +1).