CARDAMOM PLANTERS' ASSOCIATION COLLEGE

Pankajam nagar, Bodinayakanur

DEPARTMENT OF COMPUTER SCIENCE

Subject: Discrete Mathematics-I

Concept: Functions

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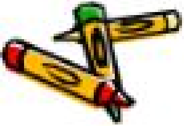
Functions

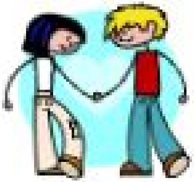
Domain and Range

Functions vs. Relations

- A "relation" is just a relationship between sets of information.

- A "function" is a well-behaved relation, that is, given a starting point we know exactly where to go.



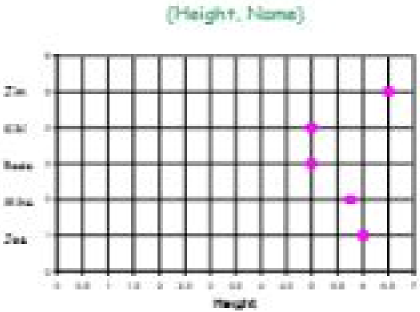


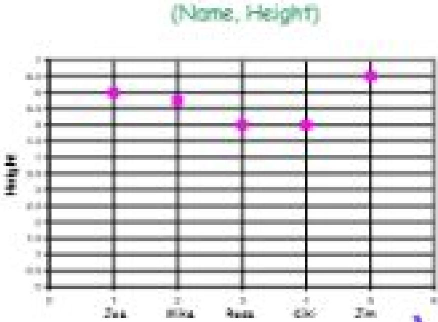
Example

- People and their heights, i.e. the pairing of names and heights.
- We can think of this relation as ordered pair:
    - (height, name)
- Or
    - (name, height)

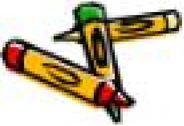
Example (continued)

Name	Height
Joe=1	6'=6
Mike=2	5'9"=5.75
Rose=3	5'=5
Kiki=4	5'=5
Jim=5	6'6"=6.5





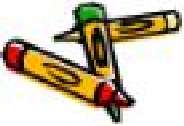
- Both graphs are relations
- (height, name) is not well-behaved .
- *Given a height there might be several names corresponding to that height.*
- How do you know then where to go?
- For a relation to be a function, there must be *exactly* one y value that corresponds to a given *x* value.



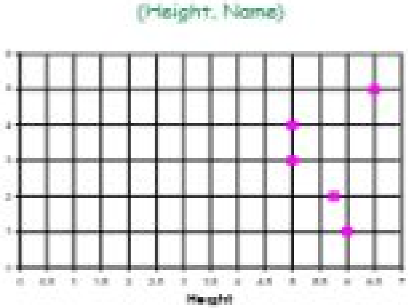
Conclusion and Definition

- Not every relation is a function.
- Every function is a relation.
- Definition:

> Let X and Y be two nonempty sets.
> A function from X into Y is a relation that associates with each element of X exactly one element of Y.



- Recall, the graph of (height, name):



What happens at the height = 5?

Vertical-Line Test

- *A set of points in the xy-plane is the graph of a function if and only if every vertical line intersects the graph in at most one point.*

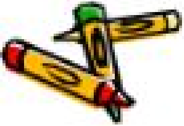
Representations of Functions

- Verbally
- Numerically, i.e. by a table
- Visually, i.e. by a graph
- Algebraically, i.e. by an explicit formula

- Ones we have decided on the representation of a function, we ask the following question:

- What are the possible x-values (names of people from our example) and y-values (their corresponding heights) for our function we can have?

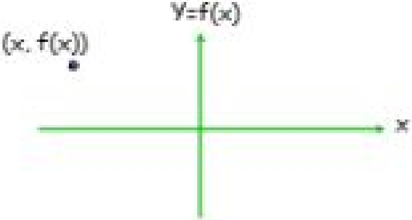
- Recall, our example: the pairing of names and heights.
    - x=name and y=height
- We can have many names for our x-value, but what about heights?
- For our y-values we should not have 0 feet or 11 feet, since both are impossible.
- Thus, our collection of heights will be greater than 0 and less that 11.



- We should give a name to the collection of possible x-values (names in our example)
- And
- To the collection of their corresponding y-values (heights).

- Everything must have a name ☺

- Variable x is called independent variable
- Variable y is called dependent variable
- For convenience, we use f(x) instead of y.
- The ordered pair in new notation becomes:
  - $(x, y) = (x, f(x))$



Domain and Range

- Suppose, we are given a function from X into Y.
- Recall, for each element x in X there is exactly one corresponding element y=f(x) in Y.
- This element y=f(x) in Y we call the image of x.
- The domain of a function is the set X. That is a collection of all possible x-values.
- The range of a function is the set of all images as x varies throughout the domain.

Our Example

- Domain = {Joe, Mike, Rose, Kiki, Jim}

- Range = {6, 5.75, 5, 6.5}

More Examples

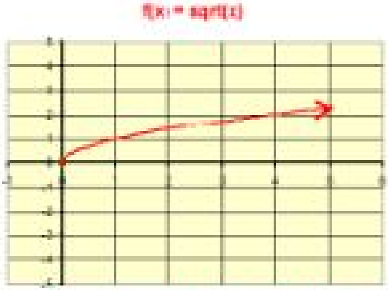
- Consider the following relation:

$$f(x) = \sqrt{x}$$

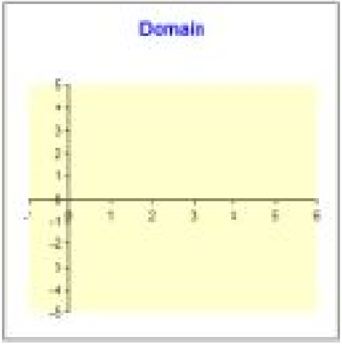
- Is this a function?
- What is domain and range?

Visualizing domain of

$$f(x) = \sqrt{x}$$

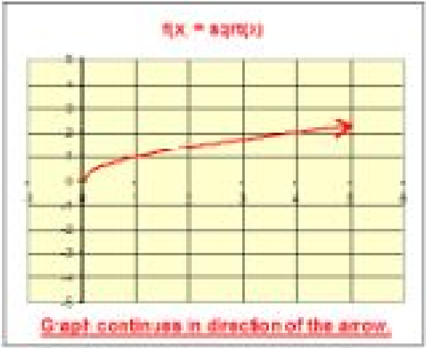


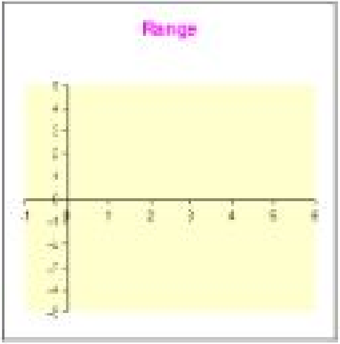
Graph continues in direction of the arrow.



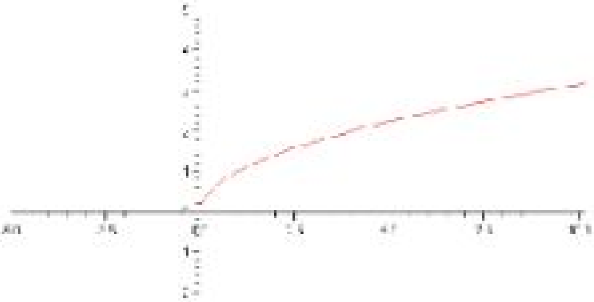
Visualizing range of

$$f(x) = \sqrt{x}$$





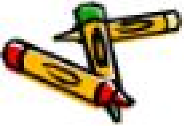
$$f(x) = \sqrt{x}$$



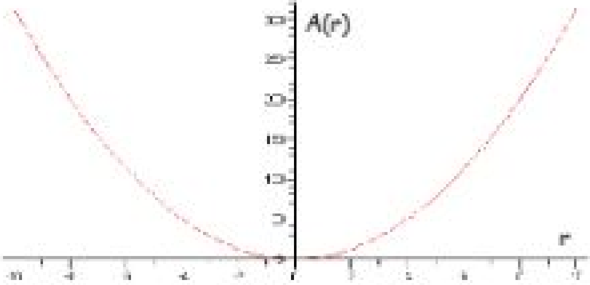
- Domain = [0, ∞)   Range = [0, ∞)

More Functions

- Consider a familiar function.
- Area of a circle:
  - $A(r) = \pi r^2$
- What kind of function is this?
- Let's see what happens if we graph $A(r)$.



Graph of $A(r) = \pi r^2$



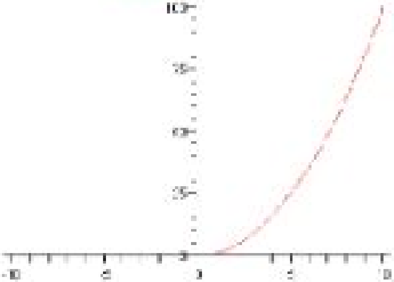
- Is this a correct representation of the function for the area of a circle???????
- **Hint:** Is domain of $A(r)$ correct?

Closer look at $A(r) = \pi r^2$

- Can a circle have $r \leq 0$?
- NOOOOOOOOOOOOO

- Can a circle have area equal to 0 ?
- NOOOOOOOOOOOOO

Domain and Range of $A(r) = \pi r^2$



- Domain = $(0, \infty)$ Range = $(0, \infty)$