counting paths between vertices

Theorem :

Suppose G is a graph with vertices $\{v_1, v_2, \dots v_n\}$ and A is the $n \times n$ adjacency matrix of G.

G can be directed or undirected and multiple edges and loops are allowed.

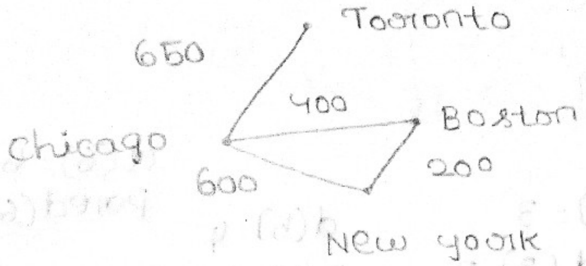
Then the $(i,j)^{th}$ entry of A^r is the number of different paths of length r from v_i to v_j

Recall :

we have seen this theorem before, when we were looking at transitive closures of relations, at least for the case of graphs without multiple edges.

Shortest Path problems

We can assign weights to the edges of graphs, for example to represent the distance between cities in a railway network



* Where does it arise in practice?

- common applications

Shortest paths in a vehicle

Shortest paths in internet routing

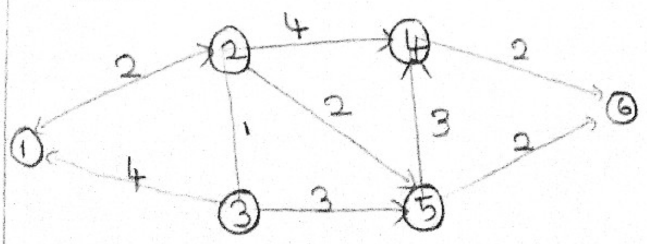
Shortest paths around MIT

- and less obvious applications, are as in the course readings

* How will we solve the shortest path problem?

- Dijkstra's algorithm.

Dijkstra's Algorithm



Exercise:

Find the shortest paths by inspection.