Combinations:

Defn: A combination is a selection of object without regart to order (or) the combinations is a un ordered collection of distinct objects.

Example:- abc is the combination of three object a, b and c

Note:

1) The number of r- combination of n distinct object is denoted by nC_r (or) $C(n,r)$ (or) $\binom{n}{r}$

2) $nC_n = nC_0 = 1$

3) $nC_r = nC_{n-r}$ (or) $C(n, n-r) \cdot C(n,r)$

Theorum:

The number of r - combinations of a set with n - elements, were n is a non-negative integer and r is an integer with $0 \le r \le n$, equals $C(n,r) = \frac{n!}{r!(n-r)!}$

Proof:

The r - permutations of a set can be obtain by 1st forming $C(n,r)$ r-combinations of the set

And then arranging the elements in each r - combinations, which can be done in $P(r,r)$ ways

Thus,
$$P(n,r) = C(n,r), P(r,r)$$
$$C(n,r)\, P(r,r) = P(n,r)$$
$$C(n,r) = \frac{P(n,r)}{P(r,r)}$$
$$C(n,r) = \frac{\frac{n!}{(n-r)!}}{\frac{r!}{(r-r)!}}$$
$$= \frac{n!}{(n-r)!} \times \frac{(r-r)!}{r!}$$
$$= \frac{n!}{(n-r)!} \cdot \frac{0!}{r!}$$
$$= \frac{n!\,(1)}{(n-r)r!}$$
$$= C(n,r) = \frac{n!}{r!(n-r)!}$$

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(1) find the value of those quantities:

a) $P(6,3)$ (b) $(3,1)$ (c) $P(3,3)$ (d) $C(8,10)$
(e) $C(5,3)$

Sol: Formula (1) $P_n r = \frac{n!}{n-r)!}$

(2) $C_n r = \frac{P!}{r!(n-r)!}$

b)

c)

d

e

(2)

3)

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a) $P(6,3) = \frac{6!}{(6-3)!} \Rightarrow \frac{6!}{3!} \Rightarrow \frac{1\times 2\times 3\times 4\times 5\times 6}{1\times 2\times 3}$

$\Rightarrow 4\times 5\times 6 = 20\times 6$

$\Rightarrow P(6,3) = \boxed{120}$

b) $P(8,1) \Rightarrow \frac{8!}{(8-1)!} \Rightarrow \frac{8!}{7!} \Rightarrow \frac{7!\,8}{7!}$

$P(8,1) = \boxed{8}$

c) $P(8,8) = \frac{8!}{(8-8)!} = \frac{8!}{0!} \Rightarrow \frac{1\times 2\times 3\times 4\times 5\times 6\times 7\times 8}{1}$

$\Rightarrow 6\times 20\times 30\times 56 \Rightarrow 120\times 30\times 56$

$= 4800\times 56 \Rightarrow P(8,8) = \boxed{40,320}$

d) $C(8,0) = \frac{8!}{0!(8-0)!} \Rightarrow \frac{8!}{0!8!} \Rightarrow \frac{8!}{8!}$

$C(8,0) = \boxed{1}$

e) $C(5,3) = \frac{5!}{3!(5-3)!} \Rightarrow \frac{5!}{3!2!}$

$= \frac{1\times 2\times 3\times 4\times 5}{1\times 2\times 3\times 1\times 2} = 2\times 5$

$= C(5,3) = \boxed{10}$

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(2) determined the value of n if $(4)(nP_3) = (n+1)P_3$

Sol:

$n! = (n-1)!\,n \quad (n+1)! = n!(n+1)$

$(n-2)! = (n-2-1)!(n-2)$

$(n-2)! = (n-3)!(n-2)$

$(4)(nP_3) = (n+1)P_3$

$nP_r = \frac{n!}{(n-r)!}$

$(4)\frac{n!}{(n-3)!} = \frac{(n+1)!}{(n+1-3)!}$

$(4)\frac{n!}{(n-3)!} = \frac{n!(n+1)}{(n-2)!}$

$(4)\frac{n!}{(n-3)!} = \frac{n!(n+1)}{(n-3)!(n-2)}$

$4 = \frac{(n+1)}{n-2}$

$4(n-2) = n+1$

$4n-8 = n+1$

$4n-n = 8+1$

$3n = 9$

$n = 9/3$

$\boxed{n=3}$

Hence proved

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3) Determined the value of n if $^{20}C_{n+2} = {}^{20}C_{2n-1}$

Sol:

Given:

n if $^{20}C_{n+2} = {}^{20}C_{2n-1}$

Formula : $nC_x = nC_y$

$\Rightarrow n = x+y$ (or) $x = y$

$\Rightarrow n+2 = 2n-1$

$2+1 = 2n-n$

$3-n \Rightarrow \boxed{n=3}$

(4) 10m How many permutation of {a,b,c,d,e,f,g}? (i) end with 'a' ? (ii) begin with 'c' ?) (iii) begin on the c and end with 'a' ? iv) c and a occupy the end

Solu :

a) The last position can be filled in only way. the remaining 6 letters can be arranged in 6 factorial ways.

∴ The total number of permutation ending with a are $= (6!) \times 1$

$= 1\times2\times3\times4\times5\times6\times1$

$= 6\times20\times6 \Rightarrow 120\times6 \Rightarrow \boxed{720}$

b) The 1st position can be filled in only one way the remaining 6 letters can be arranged in 6 factorial ways

∴ The total number of permutation Starting factorial with care $1\times(6!)$

$= 1\times2\times3\times4\times5\times6\times1 \Rightarrow 6\times20\times6$

$\Rightarrow 120\times6 = \boxed{720}$

c) begin with c and end with a {a,b,c,d,e f,g}

The first position can be filled in and last position can be filled in only one way ∴ Remaining 5 letters can be 5! ways ∴ The Total number of permutation begin with c and end with a

d)

5)

6)

c)

= (1) (5!) (1)
=) 5! =) 1x2x3x4x5 =) 6x20 =) 120

d) c and a occupy and position in 2! ways and the remaining 5 letters can be arranged in 5! ways
∴ The total number of permutation
= (5!) (2!)
= 1x2x3x4x5x2
= 6x20x2 =) 120x2 =) 240 ways

5) How many permutations of the letters ABCDEFG contain (a) the string BCD?
b) the string CFGA? (c) the string BA and GF?
d) the string ABC and DE?
e) The string ABC and CDE?

Solu:

Given: The string ABCDEFG

a) The string BCD. —> A (B C D) E F G

Taking BCD as one object, we have the following 5 objects. P(n,n) = n!

A, (BCD), E, F, G This, 5 object can be permuted in P(5,5) = 5!

= 1x2x3x4x5 =) 6x20 =) 120 ways

b) the string CFGA:

Taking CFGA as one object, we have the following B, (CFGA), D, F.

These 4 object we can be permuted in P(4,4) = 4!

= 1x2x3x4 =) 6x4 =) 24 ways.

c) The string BA and GF

Taking BA and GF as two object,

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we have the following 5 objects
(BA), C, D, E, (GF). these 5 object can be
Permuted in P(5,5) = 5!
= 1×2×3×4×5
= 6×20
= 120 ways

d) The string ABC and DE?

Taking ABC and DE as two object, we have following 4 objects.

(ABC), (DE), G, F these 4 object can be permitted in P(4,4) = 4!
= 1×2×3×4
= 6×4
= 24 ways

e) The string ABC and CDE:

even though (ABC) and (CDE) are two strings, that contain the common letter C, if we include the string (ABCDE) in the permutations, it includes both the strings (ABC) and (DE) we cannot use the letter C twice.

Hence, we have to permute the 3 objects (ABCDE), F, G this can be done in 3! = 1×2×3 = 6 ways

Hence the Result.

6) Find the number of 5 permutation of a set with 9 elements?

Sol:
Given n=9, r=5
The given is nothing but $P(n,r) = \frac{n!}{(n-r)!}$

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$$\frac{9!}{(9-5)!} = \frac{9!}{4!} \Rightarrow \frac{\cancel{1}\times\cancel{2}\times\cancel{3}\times\cancel{4}\times5\times6\times7\times8\times9}{\cancel{1}\times\cancel{2}\times\cancel{3}\times\cancel{4}}$$

$$= 5\times6\times7\times8\times9$$

$$= 30\times56\times9 \Rightarrow \boxed{15120}$$

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