UNIT: 5

BOOLEAN FUNCTIONS:

DEFN:

A Boolean expressions form for all format in n variables $x_1, x_2 \cdots x_n$ is any finite string of symbols formed in the following manner

1. 0 and 1 are boolean expressions
2. $x_1, x_2 \cdots x_n$ are boolean expressions
3. If a_1 and a_2 are Boolean expressions, when $a_1 * a_2$, $a_1 \oplus a_2$ are also Boolean expr.
4. If α is a Boolean expression then α' is also a Boolean expressions

example:

$x_1, (x_1' \oplus x_2)', (x_1' \oplus x_2) * (x_3 \oplus x_1) \cdots$ etc

NOTE:

1. Boolean expression in n variables may or may not be contain all n variable
2. The operations $\oplus, *, '$, satisfies whole the identity of a Boolean algebra, Then itis possible to define an equivalence relation on the set of Boolean expression n variables

Defn:

The two boolean forms $\alpha(x_1, x_2 \cdots x_n)$

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~~Show~~ Write that the following Boolean Algebra expressions in equivalent sum of product (SOP) canonical form in the three variables x_1, x_2, x_3. (i) $x_1 * x_2$

(i) $x_1 * x_2 \Rightarrow$ C.F of self is $2^n = 2^3 = 8$

$$LHS = (x_1 * x_2) * (x_3 \oplus x_3')$$

$$= (x_1 * x_2 * x_3) \oplus (x_1 * x_2 * x_3')$$

x_1	x_2	x_3	x_3'	$x_1 * x_2 * x_3$	$x_1 * x_2 * x_3'$	minterm
0	0	0	1	$0*(0*1)$ 0	0	M0
0	0	1	0	0	0	M1
0	1	0	1	0	0	M2
0	1	1	0	0	0	M3
1	0	0	1	0	0	M4
1	0	1	0	0	0	M5
1	1	0	1	0	1	M6
1	1	1	0	1	0	M7

$$x_1 * x_2 = (x_1 * x_2 * x_3) \oplus (x_1 * x_2 * x_3')$$

$$= M_7 \oplus M_6 \text{ (or)}$$

$$= \oplus m_{6,7} \text{ (or) } \Sigma\, 6,7$$

Product of sum of canonical form
every Boolean expression in n variables
is equivalent to a Boolean expression
consisting to the product of Maxi terms.
only such a canonical form is nun
as the product of sum of canonical forms

Find the min term in canonical form

$(x_1 + x_2) \oplus x_3$

LHS $(x_1 + x_2) \oplus x_3$

$[x_1 \oplus (x_2 + x_2')] + [x_2 \oplus (x_1 + x_1')] \oplus x_3$.

$[(x_1 \oplus x_2) + (x_1 \oplus x_2') + (x_2 \oplus x_1)(x_2 \oplus x_1') \oplus x_3$

$[x_1 \oplus x_2 \oplus x_3]\ [x_1 \oplus x_2' \oplus x_3]\ [x_2 \oplus x_1 \oplus x_3]$

$[x_2 \oplus x_1' \oplus x_3]$

x_1	x_2	x_3	x_1'	x_2'	x_3'	$(x_1 \oplus x_2 \oplus x_3)$	$x_1 \oplus x_2' \oplus x_3$	$x_2 \oplus x_1' \oplus x_3$	Mid term
0	0	0	1	1	1	0	[illegible]	1	m0
0	0	1	1	1	0	1	[illegible]	1	m1
0	1	0	1	0	1	1	0	1	m2
0	1	1	1	0	0	1	1	1	m3
1	0	0	0	1	1	1	1	1	m4
1	0	1	0	1	0	1	1	0	m5
1	1	0	0	0	1	1	1	$\oplus$	m6
1	1	1	0	0	0	1	1	1	

Max Mo + max M_2

+ max m4

(or)

mo + mo + m4

(or)

Σ 0, 1, 4.

$(x_1 + x_3') \oplus x_3$

Defn (Trom)

A boolean expression in n variables viz $x_1, x_2 \ldots x_n$ is called symmetric if inter changing any two elements results its an equivalent expression i.e) $a \oplus b = b \oplus a$

NOTE:

The defn of the symmetric boolean expression in two or three variable are inter changing then the result well be equivalent to original symmetric expression.

for example:

a) $(x_1 * x_2') \oplus (x_1' * x_2)$ b) $(x_1' * x_2') \oplus (x_1 \oplus x_2)$

c) $(x_1 * x_2' * x_3') \oplus (x_1' * x_2 * x_3') \oplus (x_1' * x_2' * x_3) \oplus$

$(x_1' * x_2' * x_3) \oplus (x_1' * x_2 * x_3)$.

Theorem:

A necessary and sub division condition that the Boolean algebra in n^{th} variables is symmetric

(i) There exist a set of no of $n_1, n_2 \ldots n_k$? the value of expression is one of any no n_i for $i, 1, 2 \ldots k$ is assigned the value one in a binary valuation process

proof:

The number $n_1 n_2 \ldots n_k$ are called statics no of the symmetric fun (the above defn of the proof of the threm) (the symmetric fun are one in a, zero and to in b, one and three in c) and b (The sum of the product of the canonical form of symmetric fun of the boolean expression of symmetric fun is

a) $(x_1 * x_2') \oplus (x_1' * x_2)$

$= [(x_1 * x_2') * (x_3 + x_3')] \oplus [(x_1' * x_2) * (x_3 \oplus x_3)]$

$= (x_1 * x_2' * x_3) \oplus (x_1 * x_2' * x_3) \oplus (x_1' * x_2 * x_3) \oplus$

$(x_1' * x_2 * x_3')$

x_1	x_2	x_3	x_1'	x_2'	x_3'	I	II	III	IV	minterm
0	0	0	1	1	1	0	0	0	0	m_0
0	0	1	1	1	0	0	0	0	0	m_1
0	1	0	1	0	1	0	0	0	1	m_2
0	1	1	1	0	0	0	0	1	0	m_3
1	0	0	0	1	1	0	1	0	0	m_4
1	0	1	0	1	0	1	0	0	0	m_5
1	1	0	0	0	1	0	0	0	0	m_6
1	1	1	0	0	0	0	0	0	0	m_7

$= m_5 + m_4 + m_3 + m_2 + m_{5,4,3,2}$

$= S_{1,2,3,4,5}$

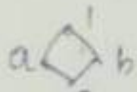
Values of Boolean expressions and Boolean functions:

Defn: Let $\alpha(x_1, x_2 \ldots x_n)$ be a boolean expression in 'n' variables and $(B, \oplus, *, 0, 1, ')$ be any boolean algebra whose elements are denoted by $a_1, a_2 \ldots a_n$.

Let $\langle a_1, a_2 \ldots a_n \rangle$ be an n tuples of B^n. We shall denote the resulting expression by $\alpha(a_1, a_2 \ldots a_n) \in B$ and call it is the value of the boolean expression $\alpha(a_1, a_2 \ldots a_n) \in B$ the process of determining of the all values is called valuation process. over the $BA(B, *, \oplus, ', 0, 1)$

NOTE: If $B = \{0, 1\}$ the valuation process over the two elements boolean algebra is called a binary valuation process.

Find the values of $x_1 * x_2 * [(x_1 * x_4) \oplus x_2' \oplus (x_3 * x_1')]$ for $x_1 = a$, $x_2 = 1$, $x_3 = b$ and $x_4 = 1$ where $(a, b) \in B$ and the boolean algebra $(B, *, \oplus, ', 0, 1)$

Sol: Let $(B, *, \oplus, ', 0, 1)$ is a boolean expression then the fig is 

given $x_1 * x_4 * [(x_1 * x_2) \oplus x_2' \oplus (x_3 * x_1')] \rightarrow ①$

given values,

$x_1 = a$, $x_2 = 1$, $x_3 = b$, $x_4 = 1$ put all the values in ①

$$x_1 * x_4 * [(x_1 * x_2) \oplus x_2' \oplus (x_3 * x_1')]$$
$$= a * 1 * [(a * 1) \oplus 1' \oplus (b * a')]$$
$$= a * [a \oplus 0 \oplus (b * b)]$$
$$= a * [a \oplus b] \quad = a$$

Alternatively the given expression is

$$x_1 * x_4 * [(x_1 * x_2) \oplus x_2' \oplus (x_3 * x_1')]$$
$$= (x_1 * x_4 * x_1 * x_2) \oplus (x_1 * x_4 * x_2') \oplus (\overline{x_1 * x_4 * x_3 * x_1'})$$

$x_1 * x_1' = 0$

$$= (x_1 * x_2 * x_4) \oplus (x_1 * x_2 * x_4) \oplus 0$$
$$= (a * 1 * 1) \oplus (a * 1 * 0) \oplus 0$$
$$= a \oplus 0 \oplus 0$$
$$= a.$$

Obtain the [illegible] $x_1 * (x_1' \oplus x_2)$, $x_1 * x_2$, $x_1 \oplus (x_1 * x_2)$ over the order pair of the two elements from the boolean algebra

Soln:

x_1	x_2	x_1'	x_2'	$x_1' \oplus x_2'$	$x_1 * (x_1' \oplus x_2)$	$x_1 * x_2$	$x_1 \oplus (x_1 * x_2)$
0	0	1	1	1	0	0	0
0	1	1	0	1	0	0	0
1	0	0	1	0	0	0	1
1	1	0	0	1	1	1	1

Then the exp of $a = x_1 * (x_1' \oplus x_2) = (x_1 * x_1') \oplus (x_1 * x_2)$

$= 0 \oplus (x_1 * x_2) = x_1 * x_2$

∴ a & b are identical then the values of x_1 and c are identical.

Defn: Boolean exp $\alpha(x_1, x_2 \ldots x_n)$ and a boolean algebra $(B, *, \oplus, ', 0, 1)$ we can obtain the values of the boolean exp for every 'n' tuples of B^n.

Let us consider a fun $f_{\alpha, B} : B^n \to B$ such that for every 'n' tuples $\langle a_1, a_2 \ldots a_n \rangle \in B^n$ the values of $f_{\alpha, B}(a_1, a_2 \ldots a_n) = \alpha(a_1, a_2 \ldots a_n)$

Find the value of the fun $f_{\alpha, B} : B^n \to B$ for $x_1 = a$, $x_2 = 1$ and $x_3 = b$ where $a, b, 1$ are the elements of the boolean algebra.

Soln: Let the sum of product of $\alpha(a, b, 1)$ is defined by $x_1' * \alpha(x_1, x_2, x_3) = (x_1' * x_2' * x_3') \oplus (x_1' * x_2 * x_3') \oplus (x_1' * x_2 * x_3) \oplus (x_1 * x_2' * x_3)$

x_1	x_2	x_3	x_1'	x_2'	x_3'	$x_1 * x_2' * x_3'$	$x_1' * x_2 * x_3'$	$x_1' * x_2$	$x_1 * x_2' * x_3$	$\alpha(x_1, x_2, x_3)$	
0	0	0	1	1	1	1	0	0	0	1	m_0
0	0	1	1	1	0	0	0	0	0	0	m_1
0	1	0	1	0	1	0	1	0	0	1	m_2
0	1	1	1	0	0	0	0	1	0	1	m_3
1	0	0	0	1	1	0	0	0	0	0	m_4
1	0	1	0	1	0	0	0	0	1	1	m_5
1	1	0	0	0	1	0	0	0	0	0	m_6
1	1	1	0	0	0	0	0	0	0	0	m_7

Then the exp of $\alpha(x_1 x_2 x_3) = \alpha(a, 1, b)$.

$= (x_1' * x_2' * x_3) \oplus (x_1' * x_2 * x_3') \oplus (x_1' * x_2 * x_3) \oplus (x_1 * x_2' * x_3)$

$= (b * 0 * a) \oplus (b * 1 * a) \oplus (b * 1 * b) \oplus (a * 0 * b)$

$= (0 \oplus (b * a) \oplus (b * b) \oplus 0 = (b * a) \oplus b = b.$