H/W

Adjaint matrix 07.02.2023

If $A = [a_{ij}]$ is a square matrix of order then the matrix

$$\begin{pmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \vdots & \vdots & & \\ A_{1n} & A_{2n} & \cdots & A_{m2} \end{pmatrix}$$

its called the adjaint of the and is denoted by adj. A (A_{ij} is the co factor of the element a_{ij})

Sence $\text{adj } A = (A_{ij})^T$

co factors of $A_{ij} = A_{ij} = (-1)^{i+1} M_{ij}$

(7m) 1) Find the adjaint of the matrix $\begin{pmatrix} 2 & 4 & -1 \\ 0 & 3 & 7 \\ 8 & 1 & 5 \end{pmatrix}$

A) a) Given $A = \begin{pmatrix} 2 & 4 & -1 \\ 0 & 3 & 7 \\ 8 & 1 & 5 \end{pmatrix}$,

Cofactors of $a_{11} = A_{11} = (-1)^{1+1} (+) \cdot \begin{vmatrix} 3 & 7 \\ 1 & 5 \end{vmatrix} = 15-7 = 8$

cofactors of $a_{12} = A_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 7 \\ 8 & 5 \end{vmatrix} = -(0-56)$

$= 56$

2)

cofactor of $a_{13} = A_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 3 \\ 8 & 1 \end{vmatrix} = 0-24$
$= -24$

cofactor of $a_{21} = A_{21} = (-1)^{2+1} \begin{vmatrix} 4 & -1 \\ 1 & 5 \end{vmatrix} = -(20+1)$
$= -21$

cofactor of $a_{22} = A_{22} = (-1)^{2+2} \begin{vmatrix} 2 & -1 \\ 8 & 5 \end{vmatrix} = -(10+8)$
$= 18$

cofactor of $a_{23} = a_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 4 \\ 8 & 1 \end{vmatrix} = -(2-32)$
$= -(-30) = 30$

cofactor of $a_{32} = A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 1 \\ 0 & 7 \end{vmatrix} = -(14-0)$
$= -14$

cofactor of $a_{33} = A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 4 \\ 0 & 3 \end{vmatrix} = (6-0)$
$= 6$

$$A_{ij} = \begin{pmatrix} 8 & 56 & -24 \\ -21 & 18 & 30 \\ 31 & -14 & 6 \end{pmatrix}$$

$\operatorname{adj} A = (a_{ij})^T$

$$\operatorname{adj} A = \begin{pmatrix} 8 & -21 & 31 \\ 56 & 18 & -14 \\ -24 & 30 & 6 \end{pmatrix}$$

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2) Find the adjaint of the matrix $\begin{pmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{pmatrix}$

sol:

Given $A = \begin{pmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{pmatrix}$

cofactor of $a_{11} = A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 2 \\ -1 & -4 \end{vmatrix} \Rightarrow -4+2 = -2$

cofactor of $a_{12} = A_{12} = (-1)^{1+2} \begin{vmatrix} -5 & 2 \\ 10 & -4 \end{vmatrix} = -(20-20) = 0$

cofactor of $a_{13} = A_{13} = (-1)^{1+3} \begin{vmatrix} -5 & 1 \\ 10 & -1 \end{vmatrix} = 5-10 = -5$

cofactor of $a_{21} = A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & -3 \\ -1 & -4 \end{vmatrix} = -(4-3) = -1$

cofactor of $a_{22} = A_{22} = (-1)^{2+2} \begin{vmatrix} 8 & -3 \\ 10 & -4 \end{vmatrix} = -32+30 = -2$

cofactor of $a_{23} = A_{23} = (-1)^{2+3} \begin{vmatrix} 8 & -1 \\ 10 & -1 \end{vmatrix} = -(8+10)$
$= -2$

cofator of $a_{31} = A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & -3 \\ 1 & 2 \end{vmatrix} = -2+3 = 1$

cofator of $a_{32} = A_{32} = (-1)^{3+2} \begin{vmatrix} 8 & -3 \\ -5 & 2 \end{vmatrix} = -(16-15) = -1$

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cofactor of $a_{33} = A_{33} = (-1)^{3+3} \begin{vmatrix} 8 & -1 \\ -5 & 1 \end{vmatrix} = 8-5 = 3$

$A_{ij} = \begin{pmatrix} 2 & 0 & -5 \\ -1 & -2 & -2 \\ 1 & -1 & 3 \end{pmatrix}$ adj $A = (A_{ij})^T$

$\text{adj } A = \begin{pmatrix} 2 & -1 & -1 \\ 0 & -2 & -1 \\ -5 & -2 & 3 \end{pmatrix}$

2)

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