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SPECTRAL INVERSE OF FUZZY MATRICES

A.Nagoor Gani[1*] and A.R.Manikandan[2]
[1,2]*P.G & Research Department of Mathematics,Jamal Mohamed College (Autonomous), Tiruchirappalli-620 020, India.*
e-mail: ganijmc@yahoo.co.in[1] and arm.mani123@gmail.com [2]

Abstract: In this paper we introduce The Moore Penroseinverse and Spectral inverse of fuzzy matrices using the structure of $M_n(F)$. We discuss the existence of group inverse and Drazin inverse of a square fuzzy matrices and The relation between group inverseand Moore- Penrose inverse of a range symmetric fuzzy matrices. The existence of group inverse for product of two square fuzzy matrices having group inverse.

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1. INTRODUCTION

The concept of fuzzy set was introduced by Zadehin[6] 1965. Ragab.M. Z. and Emam E. G.[1] introduced the determinant and adjoint of a square fuzzy matrix.Zhang, K.L.[5] introduced the nilpotent matrices over D-lattices.” Nagoorgani A. and KalyaniG.[4] introduced the Fuzzy m-norm matrices. Meenakshi A.R. and Inbam C. [3] introduced the concept of Drazin inverses of fuzzy matrices.Meenakshi A.R.[2]introduced the concept of Fuzzy matrix theory and applications.

In this paper, we first review some basic concepts of fuzzy matrices in section 2 and in section 3 the definition of Moore Penrose Inverse of Fuzzy Matrices and the generalized inverse having some of the spectral properties of the inverse of a fuzzy matrices. In section 4, the concepts of Spectral Generalized Inverse of fuzzy matrices are defined and some properties of Spectral and The Moore Penrose Inverse of Fuzzy Matricesare studied.

2. PRELIMINARIES

We consider F=[0,1] the fuzzy algebra with operation [+,·] and the standard order “≤” where a+b = max{a,b}, a·b = min{a,b} for all a,b in F.F is a commutative semi-ring with additive and multiplicative identities 0 and 1 respectively. Let M_{MN}(F) denote the set of all m×n fuzzy matrices over F. In short M_n(F) is the set of all fuzzy matrices of order n. define ‘+’ and scalar multiplication in M_n(F) as $A + B = [a_{ij} + b_{ij}]$, where A = $[a_{ij}]$ and B = $[b_{ij}]$ and cA = [ca_{ij}], where c is in [0,1], with these operations M_n(F) forms a linear space.

3. SPECTRAL INVERSE OF FUZZY MATRICES

Definition 3.1
An $m \times n$ matrix $A = (a_{ij})$ whose components are in the unit interval [0,1] is called a fuzzy matrix.
Definition 3.2
For an fuzzy matrix A of order $m \times n$, an fuzzy matrix G of order $n \times m$ is said to be g (or) ({1,2})-inverse or semi inverse of A. If AXA=A and XAX=X, where X is said to be g-inverse or a least square g-inverse of A. If AXA=A and $(AX)^T = AX$, where X is said to be g-inverse or a minimumα-norm g-inverse of A, If AXA=A and $(XA)^T = XA$, G is said to be a Moore-Penrose inverse of A in $M_n(F)$, if

$$AXA=A$$
$$XAX=X$$
$$(AX)^T = AX$$
$$(XA)^T = XA$$

and

$$A^k XA = A^k$$
$$AX=XA$$
$$A^k X = XA^k$$
$$AX^k = X^k A$$

The Moore-Penrose inverse of A is denoted by A^T.
Definition 3.3
The sequence fuzzy matrix A has a group inverse if and only if$rank A^k = rank A^{k+1}$.
Definition 3.4
Let A in $M_n(F)$ be regular if and only if A has a g-inverse. If A is regular, then a
g- Inverse of A is denoted as A^- and A$\{1\}$ is the set of all g-inverses of A satisfying $AA^-A = A$ for all A^- in A$\{1\}$.
Definition 3.5
If A in $M_n(F)$ is a range symmetric fuzzy matrix if and only if $R(A) = R(A^T)$
Let $R(M_n(F))$ be the set of all range symmetric fuzzy matrices in $M_n(F)$.
Theorem 3.1
If A is fuzzy matrix of $M_n(F)$.
(i) X exists
(ii)A^k is regular, $dimR(A) = dimR(A^k)$
(iii) Both the equations $A^k X = A$ and $XA^k = A$ can be solved for X
Proof:
(i)⇒(ii) If X exists

$$A = A^k X = XA^k$$
$$A = XA^k$$
$$dimR(A) = dimR(XA^k)$$
$$\leq dimR(A^k)$$

For a pair of fuzzy matrices X and A, if the product XA is defined then

$$dimR(XA) = dimR(X) \cdot A \subseteq dimR(A)$$
$$dimR(A) \subseteq dimR(A^k)$$

$$A = A^k X \Rightarrow dimR(A) = dimR(A^k X)$$
$$\subseteq R(A^k)$$
$$dimR(A) \subseteq dimR(A^k)$$

Therefore A is regular

(ii)⇒(iii) If A^k is regular

$$A^k\{1\} \neq \emptyset$$
$$R(A) = R(A^k)$$
$$A = A(A^k)^- A^k \text{ for all } (A^k)^- \text{ in } A^k\{1\}$$

For any $(A^k)^-$ the fuzzy matrix $(A^k)^- A$ is a solution for the fuzzy matrix equation

$$A^k X = A$$

and the fuzzy matrix $A(A^k)^-$ is a solution for the fuzzy matrix equation

$$XA^k = A$$

(iii) ⇒(i)

Let U and V be fuzzy matrix of$M_n(F)$.

$$A^k X = A = XA^k = A$$

For $Y = VAU$

$$AUA = (VA^k)UA$$
$$= V(A^k U)A$$
$$= VA^k$$
$$AUA = A \qquad (3.1)$$
$$AVA = AV(A^k U)$$
$$= A(VA^k)U$$
$$= A^k U$$
$$AVA = A \qquad (3.2)$$

Using (3.1) and (3.2)

$$\Longrightarrow AY = A(VAU)$$
$$= (AVA)U$$
$$= AU$$
$$= VA^k U$$
$$= VA$$
$$= V(AUA)$$
$$= (VAU)A$$
$$AY = YA$$
$$\Longrightarrow YAY = (VAU)A(VAU)$$
$$= V(AUA)VAU$$
$$= V(AVA)U$$
$$= VAU$$
$$YAY = Y$$
$$\Longrightarrow AYA = A(VAU)A$$
$$= (AVA)UA$$
$$= AUA$$
$$AYA = A$$

Hence Y=X is the group inverse of Ain $M_n(F)$.

Theorem 3.2

If A is fuzzy matrix of $M_n(F)$, then A^+ exists. If A is range symmetric ⟺ $A^{\#T}$ exists,

$$A^{\#T} = A^{T\#}$$

Proof:

If A is range symmetric

$R(A) = R(A^{T\#})$

$\Longrightarrow R(A) \subset R(A^{T\#})$

$\Longrightarrow A = XA^{T\#}$

$= XA^{T\#}(A^{T\#})^{-}A^{T\#}$

$A = A(A^{T\#})^{-}A^{T\#}$ for all $(A^{T\#})^{-}$ in $A^{T}\{1\}$

Since $(A^{T\#})^{+}$ is a g-inverse of $A^{T\#}$

$A = A(A^{T\#})^{+}A^{T\#}$

$A = A(A^{T})^{T}A^{T\#}$

$= AAA^{T\#}$

$A = A^{2}A^{T\#}$

$R(A^{T\#}) \subseteq R(A) \Rightarrow A^{T\#}A^{-}A$ for all A^{-} in A$\{1\}$

$=A^{T\#}(A^{+})^{T}A$

$=A^{T\#}A^{T\#}A$

A$=(A^{T\#})^{2}A$

A$A^{T\#} = AA^{+} = A((A^{T\#})^{2}A)$

$A^{+}A = A^{T\#}A = (A^{T\#}A^{2}(A^{T\#}$

$= A(A^{2})^{T\#}A$

Since $A^{T\#}A$ is symmetric

$=$A$A^{T\#}$

$=$AA^{+}

Hence A$A^{+} = A^{+}A$for all A^{+} in $A\{1,2\}$

$\Longrightarrow A^{T\#} = A^{+} = A^{\#T}$

Hence $A^{\#T}$ exists $A^{\#T} = A^{T\#}$

Conversely, if $A^{\#T}$ exists, then $A^{\#T} = (A^{\#T})^{2}A$

$\Longrightarrow R(A^{\#T}) = R((A^{\#T})^{2}A) \subseteq R(A)$

Since $A^{\#T} = A^{T\#} \Longrightarrow$ R$(A^{T\#}) \subseteq$ R(A)

A$=A^{2}A^{\#T} \Longrightarrow$ R(A) $\subseteq R(A^{\#T})$

$=$ R$(A^{T\#})$

$\Longrightarrow$ R(A) $\subseteq R(A^{T\#})$

Hence A is range symmetric,$A^{\#T}$ in A$\{1,2\}$ and $A^{\#T} = A^{T\#} \Longrightarrow A^{+} = A^{\#T}$

Hence A^{+} is exists.

Theorem 3.3

Let A in M_{n}(F), If $A^{\#}$ exists,

(i) $A^{\#} = A^{\#}$AA$=$AA$A^{\#}$ (or) $A^{\#} = A^{\#}A^{2} = A^{2}A^{\#}$

(ii) $A^{\#} = A(A^{3})^{(1)}A$, where $(A^{3})^{(1)}$ an element of $A^{3}\{1\}$

Proof:

(i)$R(A^{\#}) = R(A^{\#}A) \subseteq R(A) = R(AA)$

$\Longrightarrow$ R$(A^{\#}) \subseteq$ R(AA)

$\Longrightarrow A^{\#} = A^{\#}(AA)^{-}AA$for all (AA) in A$\{1\}$

$\Longrightarrow A^{\#} = A^{\#}(AA)AA$
$= A^{\#}AA$
$A^{\#} = A^{\#}A^{2}$
$\Longrightarrow A^{\#} =$ AA $(AA)^{-}A^{\#}$
$= AA(AA)A^{\#}$
$=AAA^{\#}$
$A^{\#} = A^{2}A^{\#}$
(ii) Let X=$AA^{3}A$
Claim: X= $A^{\#}$
$\Longrightarrow A^{\#}XA^{\#} = A^{\#}(AA^{3}A)A^{\#}$
$=A^{\#}(AAAAA)A^{\#}$
$= (A^{\#}AA)A(AAA^{\#})$
$= (A^{\#}A^{2})A(A^{\#}A^{2})$
$= A^{\#}AA^{\#}$
$A^{\#}XA^{\#} = A^{\#}$
$\Longrightarrow XA^{\#}X = X(A^{\#}A^{\#}A^{\#}A^{\#}A^{\#})X$
$= (X(A^{\#})^{2})\ A^{\#}((A^{\#})^{2}X)$
$= XA^{\#}X$
$XA^{\#}X = X$
Also $A^{\#}X = A^{\#}(AAAAA)$
$= (A^{\#}AA)AAA$
$= A^{\#}AAA$
$A^{\#}X = A^{\#}A = XA$is symmetric
Therefore $XA = AA \Rightarrow XA$ is symmetric.
Thus X in A{1}, Hence $A^{\#}$ exists and $A^{\#} = X = A(A^{3})^{(1)}A$

4. CONCLUSION

In this paper, The Moore Penrose inverse and Spectral inverse of Fuzzy Matricesand its propertiesarediscussed. Numerical examplesare given to clarify the developed theory and the proposed Spectral Inverse of Fuzzy Matrices.

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