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Some Results on Fuzzy Matrix 1, 2, P-Norms

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Abstract: *In this paper, the definition of fuzzy matrix norm in $M_n(F)$ and fuzzy matrix operator norm is given. Some of its properties are discussed. Fuzzy matrix norm, 1-norm, 2-norm, p-norm are also defined.*

AMS Subject Classification: 03E72, 15B15

Keywords:*matrix norm, Fuzzy matrix, Fuzzy norm, Fuzzy norm matrix, determinant of a square fuzzy matrix*

I. Introduction

The concept of fuzzy set was introduced by [10] Zadeh in 1965. A. White introduced the concept of 2-Banach Space [8]. Also those definitions are restricted to the fuzzy normed linear spaces and Convergence of power of a controllable fuzzy matrices .Felbin[2] and L.J. Xin [9],1995,Ragab.M. Z. and Emam E. G.[3] introduced the determinant and adjoint of a square fuzzy matrix. Meenakshi A.R. and Cokilavany R. [4] introduced the concept of fuzzy 2-normed linear spaces. Nagoorgani A. and Kalyani G. [6] introduced the properties of fuzzy m-norm matrices. In Nagoorgani A. and Kalyani G. [1] T.Bag introduced theFundamental Theorem in Felbins Type Fuzzy Normed Linear Space. [7]Nagoorgani A. and Manikandan A.R, introduced the concept of Integrals over Fuzzy Matrices. [5]Madhumangal Pal and RajkumarPradhan Also those definitions are restricted to theTriangular Fuzzy Matrix Norm. In this paper, we introduce the concept of fuzzy norm matrices. The purpose of the introduction is to explain 1,2,p- norms and its properties are discussed.

II. Preliminaries

We consider F=[0,1] the fuzzy algebra with operation [+,·] and the standard order "≤" where a+b = max{a,b}, a·b = min{a,b} for all a,b in F.F is a commutative semi-ring with additive and multiplicative identities 0 and 1 respectively. Let M_{MN}(F) denote the set of all m×n fuzzy matrices over F. In short M_n(F) is the set of all fuzzy matrices of order n. define '+' and scalar multiplication in M_n(F) as A + B $= [a_{ij} + b_{ij}]$, where A = $[a_{ij}]$ and B = $[b_{ij}]$ and cA = $[ca_{ij}]$, where c is in [0,1], with these operations M_n(F) forms a linear space.

Definition 2.1

A fuzzy matrix (FM) of order $m \times n$ isdefined as $A = [a_{ij}]$ or $P_A = [a_{ij}]$where a_{ij} is the membership value of the i,j-th element in A. Let $M_{mn}(F)$ denote the set of all fuzzy matrices of order $m \times n$ over F. If $m = n$ in short we write $M_n(F)$ the set of all square FMs of order n.

Definition 2.2 :

Let A be in $M_n(F)$ and α be in [0,1] such that ||A||= α or $n(P_A) = \alpha$, then the pair (P_A, α) is called a fuzzy point in $M_n(F)$ and it is denoted by P_A^α. The dual fuzzy point P_A^α is the point with norm (1- α) denoted by $P^* = P^{1-\alpha}$.

Definition 2.3 :

The set of all fuzzy points in $M_n(F)$ is gven by

$P^*(M_n(F)) = \{P_A^\alpha \mid A \in M_n(F), \quad \alpha \in [0,1]\}$

In F we follow the usual ≤ order relation correspondingly we define an order relation in $P^*(M_n(F))$ as follows

Definition 2.4:

We define $P_A^\alpha < P_B^\beta$ iff $\alpha < \beta$ and $P_A^\alpha = P_B^\beta \; iff \; A = B$ (then automatically $= \beta$)

Definition 2.5:

A fuzzy norm with respect to the fuzzy matrix norm in $M_n(F)$ is a real valued function n defined on $P^*(M_n(F)) \times P^*(M_n(F))$ to [0,1] satisfying the following conditions

(i) $n(P_A) = 0$

(ii) $n(\alpha P_A) = |\alpha| \, n(P_A)$

(iii) $n(P_A + P_B) \leq n(P_A) + n(P_B)$

Definition 2.6:

Let $P^*(M_n(F))$ be a fuzzy vector space, where $n(P_x)$ is some norm , we define the norm of the matrix $A_{mn} \; in \; M_{mn}(F)$ subordinate to the vector norm $n(P_x)$ as

$$n(P_A) = {}_{n(P_A)\neq 0}^{max}(n(P_A P_x)/n(P_x)).$$

The above norm is called sub-ordinate fuzzy matrix norm.

Let us consider the vector norm as

$$n(P_x) = \max[x_i]\,, where \; P_x = [x_i]$$

(Or)

$$n(P_x) = \max[x_{1,} x_{2,} \dots x_n]$$

(or)

$n(P_x) = \sum_{i=1}^{n} |x_i|$denoted on the set $P^*(M_n(F))$and the fuzzy matrix norm.

$$n(P_A) = \max[a_{i,j}]\,, where \; P_A = [a_{i,j}]$$

(or)

$$n(P_A) = \max[a_{11,} a_{12,} \dots a_{ij}, \dots a_{nn}]$$

$n(P_A) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}$denoted on the set $(M_n(F))$.

(i) $n(P_A P_x)$is defined for $P_A = P_x \; as \; 0$,

$n(P_A P_x) \neq 0$is defined for $P_A < P_x$

when$P_A = P_x$ both fuzzy points coincide $n(P_A P_x) = 0, \quad for \; all \; P_A, P_x \in P^*(M_n(F))$

(ii) Since $P_A, P_x \in P^*(M_n(F))$

$$n(\alpha P_A) = {}_{n(P_x)\neq 0}^{max}(n(\alpha P_A P_x)/n(P_x))$$

$$= {}_{n(P_x)\neq 0}^{max}(|\alpha| n(P_A P_x)/n(P_x))$$

$$= |\alpha|{}_{n(P_x)\neq 0}^{max}(n(P_A P_x)/n(P_x))$$

$$n(\alpha P_A) = |\alpha| n(P_A)$$

(iii) $\quad n(P_A) = {}_{n(P_x)\neq 0}^{max}(n((P_A + P_B)P_x)/n(P_x))$

$$n(P_A) = n((P_A + P_B)P_{x_m})/n(P_{x_m}))$$

Where $P_{x_m} \in P^*(M_n(F))$ maximizes the right hand side.

Define $P_A P_{x_m} = P_{y_m}$ and $P_B P_{x_m} = P_{z_m}$

$$n((P_A + P_B)P_{x_m}) = n(P_A P_{x_m} + P_B P_{x_m})$$

$n(P_A + P_B) \cap (P_{x_m}) = n(P_{y_m} + P_{z_m})$

$$\leq n(P_{y_m}) + n(P_{z_m})$$

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$n(P_A + P_B) \cap (P_{x_m}) \leq n(P_A P_{x_m}) + n(P_B P_{x_m})$

$$n(P_A + P_B) \leq \frac{n(P_A P_{x_m}) + n(P_B P_{x_m})}{n(P_{x_m})}$$

$$\leq {}_{n(P_x)\neq 0}^{max}(n(P_A P_{x_m})/n(P_{x_m})) + {}_{n(P_x)\neq 0}^{max}(n(P_B) \cap (P_{x_m})/n(P_{x_m})$$

$$n(P_A + P_B) \leq n(P_A) + n(P_B) .$$

(iv) By Definition,

$$n(P_A) \geq n(P_A P_x)/n(P_x)$$

$$n(P_A) \cap n(P_x) \geq n(P_A P_x), \forall P_x \in M_n(F)$$

(v) If $P_A, P_B \in M_n(F)$, then

$$n(P_A P_B) = {}_{n(P_x)\neq 0}^{max}(n(P_A P_B P_x)/n(P_x))$$

$$n(P_A P_B) = {}_{n(P_x)\neq 0}^{max}(n(P_A P_B P_{x_m})/n(P_{x_m}))$$

If we define $n(P_B P_{x_m}) = P_{z_m}$ in $P^*(M_n(F))$

$$n(P_A P_B P_x) = n(P_A P_{z_m})$$

$$\leq n(P_A)n(P_{z_m})$$

$$\leq n(P_A)n(P_B P_{z_m})$$

$$\leq n(P_A)n(P_B)n(P_{z_m})$$

$$n(P_A P_B P_x) \leq n(P_A)n(P_B)n(P_{z_m})$$

Hence $n(P_A P_B) = n(P_A P_B P_{x_m})/n(P_{x_m})$

$$\leq n(P_A)n(P_B)n(P_{x_m})/n(P_{x_m})$$

$$n(P_A P_B) \leq n(P_A)n(P_B)$$

This n is a fuzzy norm with respect to the fuzzy matrix norm in $M_n(F)$ and hence $M_n(F)$ is a fuzzy norm with respect to the fuzzy matrix norm.

Definition 2.7:

A fuzzy matrix norm n on a the set $M_n(F)$ is a norm, with the additional property that

$n(P_A P_B) \leq n(P_A)n(P_B)$ for all P_A, P_B in $M_n(F)$.

Definition 2.8:

Let n_α be a vector norm on $P^*(M_n(F))$, and n_β a vector norm on $P^*(M_n(F))$, for $P_A \in M_n(F)$.

We define $n(P_A) = n_{\alpha,\beta}(P_A) = {}_{n(P_x)\neq 0}^{max}(n_\beta(P_A P_x)/n_\alpha(P_x)$ for all $, \beta \in [0,1]$.

Sub-ordinate fuzzy matrix norm with respect to different vector norm. They be general l_1 norm and l_2 norm respectively. If $\alpha = \beta$ then the above fuzzy norm is called $\alpha - norm$

III. Fuzzy Matrix 1-Norm

The vector 1-Norm is given by

$$n_1(P_x) = \sum_{i=1}^{n} |x_i|$$

Subordinate to the vector 1-norm is the fuzzy matrix 1-norm .

$$n_1(P_A) = \max_j(\sum_i |a_{i,j}|)$$

The fuzzy matrix 1-norm is the maximum of the column sums in $M_n(F)$ and let $m \times n$ matrix P_A in $M_n(F)$ be represented.

$$P_A = [P_{A_1}, P_{A_2}, \dots, P_{A_n}]$$

Which implies

$$P_A P_x = [P_{A_1}, P_{A_2}, \dots, P_{A_n}] P_x$$

$$P_A P_x = \sum_{i=1}^{n} P_{A_k} P_{x_k}$$

$$n_1(P_A P_x) = n_1(\sum_{i=1}^{n} P_{A_k} P_{x_k})$$

$$\leq \sum_{i=1}^{n} |P_{x_k}| \cap (P_{x_k})$$

$$\leq \max_j n(P_{A_j})(\sum_{i=1}^{n} P_{x_k})$$

$$n_1(P_A P_x) = \max_j n(P_{A_j}) n(P_x)$$

IV. Fuzzy Matrix 2-Norm

The vector 2-Norm is given by

$$n_2(P_x) = (\sum_{i=1}^{n} |x_i|^2)^{1/2} = (< x, x >)^{1/2}$$

Subordinate to the vector 2-norm is given by

$n_2(P_A) = P_A * P_A$

The Fuzzy 2-Norm is called the spectral norm. The arbitrary $m \times n$ fuzzy matrix P_A in $M_n(F)$ and $P_A * P_A$is $n \times n$ and Hermitian, the eigen values of $P_A * P_A$ are real valued. Also $P_A * P_A$ is at least positive fuzzy semi-definite.

Since $P_x * (P_A * P_x) * P_x = P_A P_x * P_A P_x$ for all $P_x \in P^*(M_n(F))$

Hence the eigen values of $P_A * P_A$ are both real valued and non-negative.

Denote them as

$$0 \leq \sigma_1^2 \geq \sigma_2^2 \geq \sigma_3^2 \geq \dots . \sigma_n^2 \geq 1$$

Corresponding to these eigenvalues are n orthonormal and eigen vectors $U_{1,} U_2, U_3 \dots . U_n$

$$(P_A * P_A) U_k = (\sigma_k^2) U_k$$

The n eigenvectors form the columns of a unitary $n \times n$ fuzzy matrix U that diagonalizes fuzzy matrix

$P_A * P_A$in $P^*(M_n(F))$.

$$P_x = \sum_{i=1}^{n} P_{c_k} U_k \text{ where} P_{c_k} \text{ in } M_n(F)$$

$$P_A * P_A P_x = P_A * P_A \sum_{i=1}^{n} P_{c_k} U_k$$

$$P_A * P_A P_x = \sum_{i=1}^{n} P_{c_k} \sigma_k^2 U_k$$

$$(n_2(P_A P_x))^2 = (P_A P_x) * P_A P_x$$

$$= P_x * (P_A * P_A P_x)$$

$$= (\sum_{k=1}^{n} P_{c_k} * U_k)(\sum_{j=1}^{n} P_{c_j} \sigma_j^2 U_j)$$

$$= \sum_{k=1}^{n} \left|P_{c_k}\right|^2 \sigma_k^2 U_k$$

$$\leq \sigma_1^2 \left(\sum_{k=1}^{n} \left|P_{c_k}\right|^2 U_k\right)$$

$$(n_2(P_A P_x))^2 = \sigma_1^2 n_2(P_x)^2$$

V. Fuzzy Matrix p-norm

For $m \times n$ fuzzy matrix P_A in $M_n(F)$

$n_p(P_A P_x) \leq n_p(P_A) n_p(P_x)$for all P_x in $M_n(F)$

$$n_p(P_A P_B P_x) = n_p(P_A(P_B P_x))$$

$$\leq n_p(P_A) n_p((P_B P_x)$$

$$n_p(P_A P_B P_x) \leq n_p(P_A) n_p(P_B) n_p(P_x)$$

$$\frac{n_p(P_A P_B P_x)}{n_p(P_x)} = n_p(P_A) n_p(P_B)$$

$$n_p(P_A P_B) \leq n_p(P_A) n_p(P_B)$$

Theorem 5.1

For the fuzzy matrices $P_A, P_B \in M_n(F)$ and $P_v, P_w \in P^*(M_n(F))$

(i) $n_p(P_A) = n_p(P_{A^T})$

(ii) $n_p(P_A) = n_p(P_{A^T})$

(iii) $n_p(P_A P_{A^T}) = n_p(P_{A^T} P_A) = (n_p(P_A))^2$

(iv) $|\langle P_A P_v, P_w \rangle| \leq n_p(P_A) n_p(P_v) n_p(P_w)$

Proof

(i) For any $P_v \in P^*(M_n(F))$

$$(n_p(P_A P_v))^2 = |\langle P_A P_v, P_A P_v \rangle|$$

$$n_p(P_A P_v) = |\langle P_v P_{A^T}, P_A P_v \rangle| = n_p(P_v) n_p(P_{A^T} P_A P_v)$$

$$n_p(P_A P_v) = (n_p(P_{A^T} P_A) n_p(P_v))^{\frac{1}{2}}$$

Therefore $n_p(P_A) = n_p(P_{A^T})$

(ii) We have $(n_p(P_A))^2 \leq n_p(P_{A^T} P_A) \leq n_p(P_{A^T}) n_p(P_A) = (n_p(P_A))^2$

Therefore $(n_p(P_A))^2 = n_p(P_{A^T} P_A)$

(iii) $|\langle P_A P_v, P_w \rangle| \leq n_p(P_A P_v) n_p(P_w)$

$$n_p(P_A P_v) \leq (n_p(P_{A^T} P_A))^{\frac{1}{2}} n_p(P_v) \leq n_p(P_A) n_p(P_v)$$

$$|\langle P_A P_v, P_w \rangle| \leq n_p(P_A) n_p(P_v) n_p(P_w)$$

VI. References

[1] T.Bag.Some Fundamental Theorem in Felbins Type Fuzzy Normed Linear Space. IJMSC, Vol. 1.No, 2, 2011.

[2] Ragab.M. Z. and Emam E. G. The determinant and adjoint of a square fuzzy matrix, An international journal of Information Sciences-Intelligent systems, Vol 84, 1995, 209-220.

[3] Ragab.M. Z. and Emam E. G. The determinant and adjoint of a square fuzzy matrix, An international journal of Information Sciences-Intelligent systems, Vol 84, 1995, 209-220.

[4] Meenakshi A.R. and Cokilavany R., On fuzzy 2-normed linear spaces, The Journal of fuzzy mathematics, volume 9(No.2) 2001 (345-351)

[5] Madhumangal Pal and RajkumarPradhan., Triangular Fuzzy Matrix Norm and Its Properties, The Journal of Fuzzy Mathematics Vol. 25, No. 4, 2017

[6] Nagoorgani A. and Kalyani G. On Fuzzy m-norm matrices. Bulletin of Pure and Applied Sciences. Vol. 22E(No.1) 2003; P. 1-11

[7] Nagoorgani A. and Manikandan A.R. Integrals over Fuzzy Matrices, international journal of maths. Sci & Engg. Appls, Vol.7 No. 6 (2013) pp 225-232

[8] A. White, 2-Banach Space, Math. Nachr 42(1969),43-60

[9] L.J. Xin Convergence of power of a controllable fuzzy matrices, Fuzzy sets and system, 45(1992)313-319

[10] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338-353.

VII. Acknowledgments

In this paper, the 1,2,p-norm- matrices and its properties are suggested in fuzzy environment. A numerical example is given to clarify the developed theory and the proposed 1,2,p-norms with fuzzy matrix.