MATHEMATICAL MODELING ON NON-LINEAR DYNAMICS

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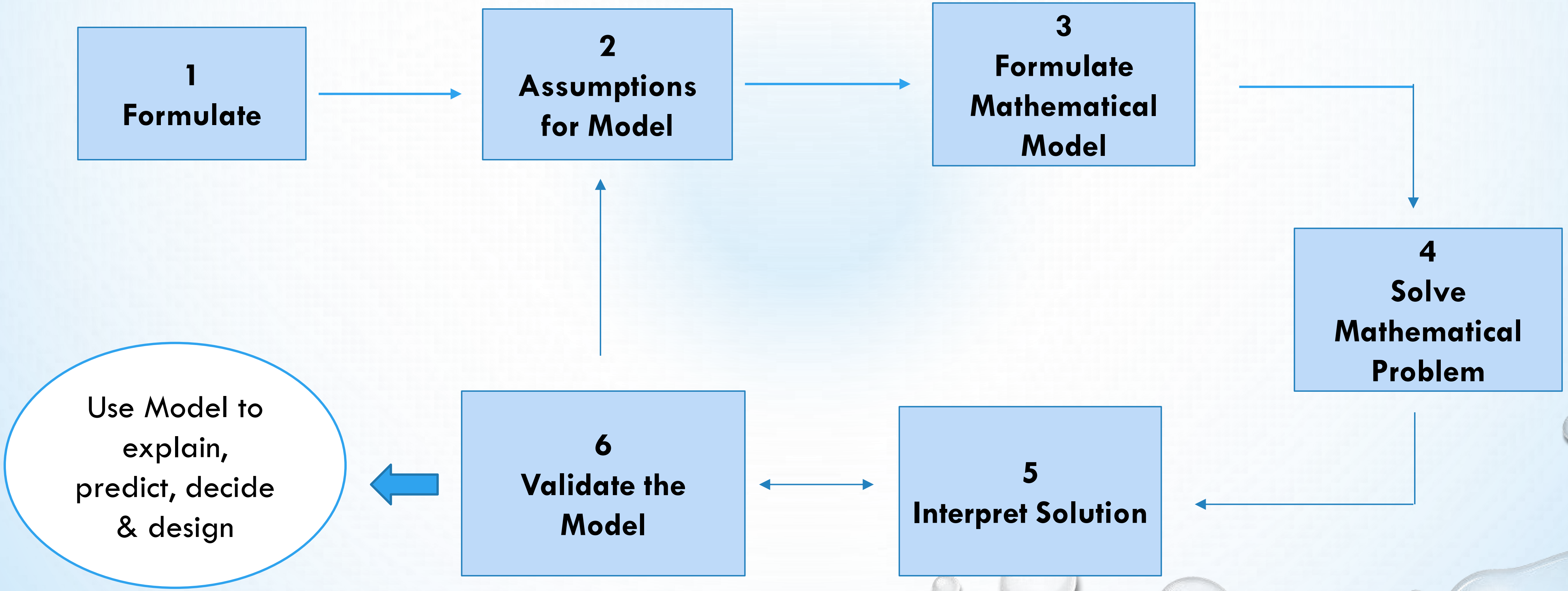
Mathematical Modelling

Origin:

The early development of modelling is due to physical sciences

1. Klamin (1971) proposed Five stage model for solving process.
2. Lin (1976) Added Evolution of modelling process as Sixth stage.



APPLICATION OF MODELLING IN DAY TO DAY LIFE

- Mathematical Modelling has a vast application in developing sciences,
- Modelling are used in various field to determine, predict and to conclude their problems relating their field.
- Some of the fields are

❖ Weather prediction

❖ Meteor path computation

❖ Outbreak of diseases

❖ Space mission

❖ Evolution of particles

<u>Modelling by solving Differential Equation</u>

To solve Differential equation we need to check some conditions

- Linear (or) Non- Linear Equation
- Dynamical (or) Static equation
- Initial Value (or) boundary Value Problem

<u>Linear Equation:</u>

The differential equation which is independent of other variable with respect to time is said to be Linear Equation.

General Differential Equation of Form,

$$\frac{d^n x}{dt} + A_1(t)\frac{d^{n-1}x}{dt^{n-1}} + A_2(t)\frac{d^{n-2}x}{dt^{n-2}} + \cdots + A_{n-1}(t)\frac{dx}{dt} + A_n x = 0$$

<u>Example for Linear Differential Equation:</u>

$$\frac{dx}{dt} = -k_1 x$$

Non- Linear Equation:

The differential equation which includes product of dependent variables or their derivatives are called Non- Linear differential equations.

Example:

$$\left(\frac{dx}{dt}\right)^2 = b_1x + C$$

Dynamical System:

A system of equation that changes with respect to time is said to Dynamical equations.

Initial Value Problem (IVP):

The problem of finding a function y(x), when we know its derivative and initial value $y_0(x_0)$ is called initial value problem.

Boundary Value Problem (BVP):

Boundary value problem the conditions are not directly known, but the boundary conditions are included in the value of the variable or value for derivatives of the variable.

Application of Non – Linear Differential Equation

Lotka – Volterra Equation

It is pair of First order Non- Linear differential equation,

Stage 1:

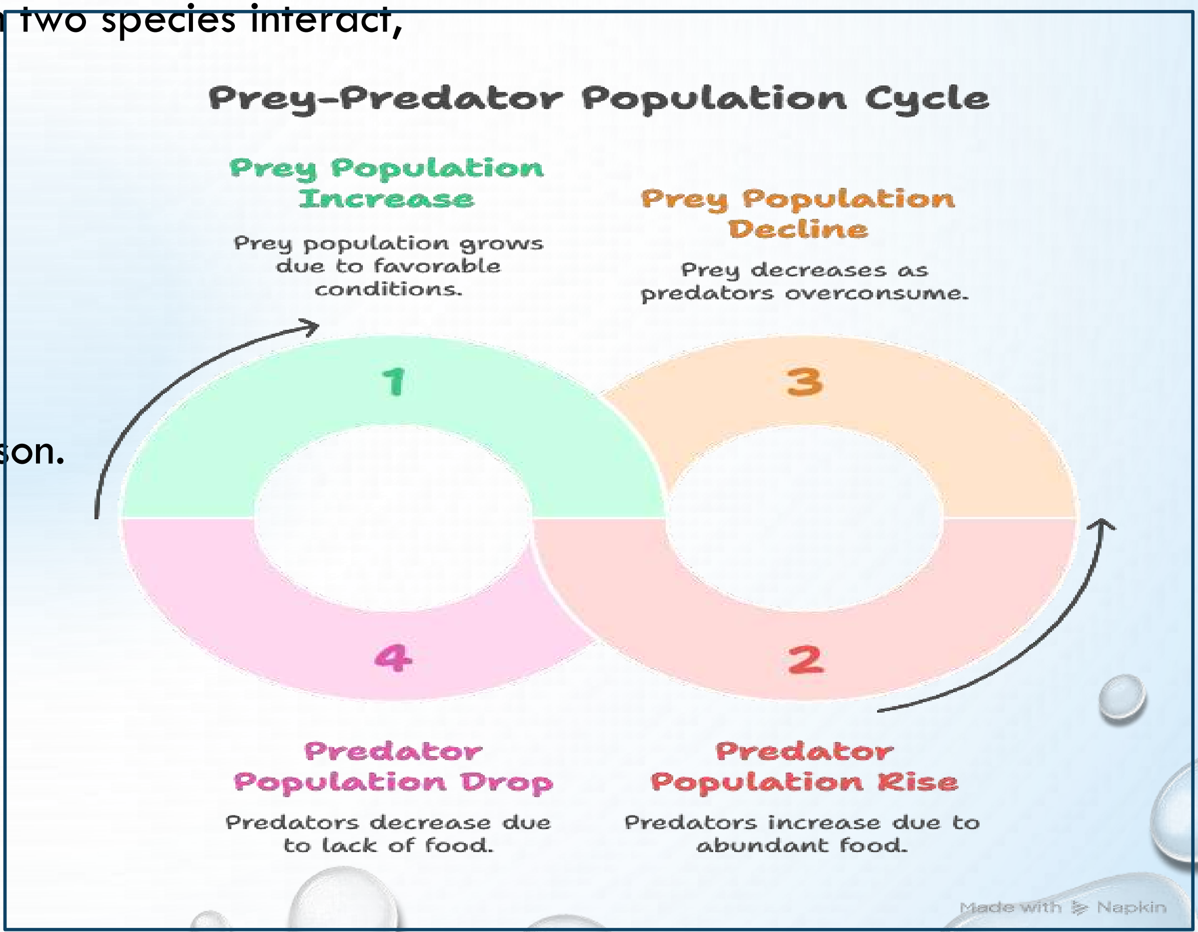
We need to describe the dynamics of biological system in which two species interact,

- One as a predator and
- Other as prey.

Stage 2:

The basic Assumption to solve this problem,

- ❖ The infection is transmitted by infected person to normal person.
- ❖ The population is very large and is Exponentially increasing.
- ❖ Consider no death due to any other cause.
- ❖ Some people got self immune to infection.



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STAGE 3:

FORMULATION OF MATHEMATICAL PROBLEM

- **PREY EQUATION:**

$$\frac{dx}{dt} = \alpha x - \beta xy$$

X - IS THE NUMBER OF HEALTHY POPULATION OF THE PREY

Y - IS THE NUMBER OF HEALTHY POPULATION OF PREDATOR

t - REPRESENTS TIME

αx – REPRESENTS EXPONENTIAL GROWTH OF PREY POPULATION

βxy – THE RATE AT WHICH HEALTHY PREY MEET AN PREDATOR AT TIME T

- **PREDATOR EQUATION:**

$$\frac{dy}{dt} = \delta xy - \gamma y$$

X - IS THE NUMBER OF HEALTHY PREY POPULATION

Y - IS THE NUMBER OF HEALTHY POPULATION OF PREDATOR

t - REPRESENTS TIME

δxy – REPRESENTS THE GROWTH RATE OF PREDATORS DUE TO CONSUMING PREY.

γy – REPRESENTS THE NATURAL DEATH RATE OF PREDATORS IN ABSENCE OF PREY

Stage 4:

Solve Mathematical Problem

This corresponds to eliminating time from the two differential equations to produce single differential equation.

$$\frac{dx}{dt} = \alpha x - \beta xy \qquad \frac{dy}{dt} = \delta xy - \gamma y$$

$$\frac{dy}{dx} = -\frac{x}{y}\,\frac{\delta x - \gamma}{\beta y - \alpha}$$

$$\frac{\beta y - \alpha}{y}\,dy + \frac{\delta x - \gamma}{x}\,dx = 0$$

$$\int \frac{\beta y - \alpha}{y}\,dy + \frac{\delta x - \gamma}{x}\,dx$$

$$V = \delta x - \gamma \ln(x) + \beta y - \alpha \ln(y)$$

V is a constant quantity depending on the initial conditions and conserved on each curve.

<u>STAGE 5:</u>

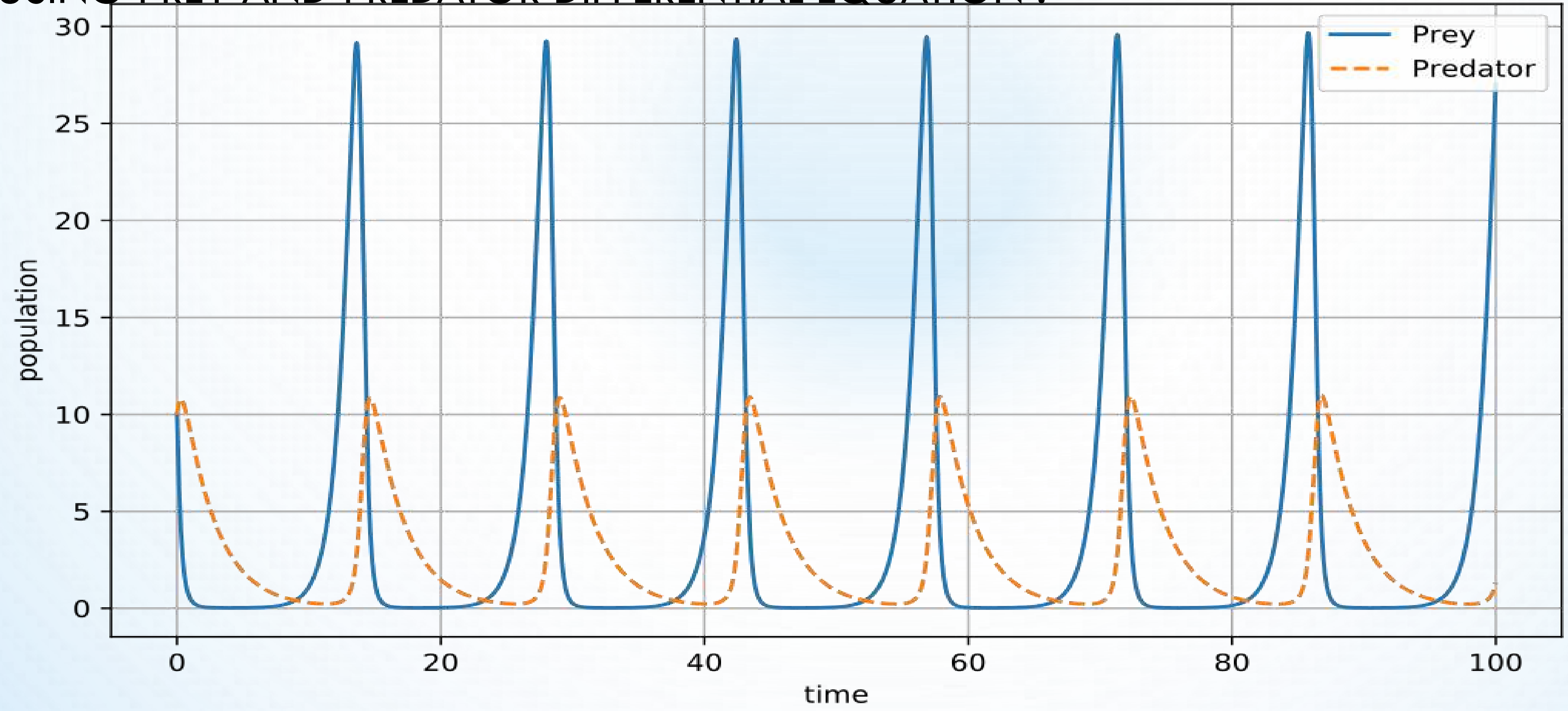
<u>INTERCEPTION</u>

(i) Suppose there are two species **PREY** as ‘x’ and **PREDATOR** as ’y’.

initially we have ***PREY ‘s Population (x) = 10 and Predator’s population (y) = 10***

and the parameters are $\boldsymbol{\alpha} = \mathbf{1.1}, \boldsymbol{\beta} = \mathbf{0.4}, \boldsymbol{\delta} = \mathbf{0.1}, \boldsymbol{\gamma} = \mathbf{0.4}$

- USING PREY AND PREDATOR DIFFERENTIAL EQUATION .



STAGE 6:

Validate the Solution

Equilibrium solutions to the Differential Equations

By solving the equation of prey and predator the solution for x and y giving the following two solutions

1. $x = 0 \; and \; y = 0$

The above solution shows that if both populations are extinct, then they will continue to be extinct until an outside factor can change that.

2. $x = \frac{\alpha}{\beta} \; and \; y = \frac{\gamma}{\delta}$

The above solution describes an equilibrium where the population of the prey is equal to the ratio of the constant $\alpha \; and \; \beta$, and the population of the predator is of ratio $\delta \; and \; \gamma$

THANK YOU