RETRIAL QUEUE IN A SERVICE FACILITY SYSTEM - MARKOV DECISION MODELS

A SYNOPSIS
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SYNOPSIS

Introduction

The Markov decision model is a versatile and powerful tool for analyzing probabilistic sequential decision process with an finite/ infinite planning horizon. The models in this thesis are an outgrowth of the Markov model and dynamic programming. The basic ideas of dynamic programming are states, the principle of optimality and functional equations.Research on Markov Decision Process has expanded at a fast rate and a powerful technology has been developed. However, in that period, relatively little effort was put into applying the quite useful Markov decision model to practical problems. The Markov decision model has many potential applications in inventory control, maintenance, manufacturing, computer network and tele-communication among others.

Literature review

Berman, O., et al. (1993)[7] analyzed a deterministic system where inventory is depleted at the demand rate when there is no queue and at the service rate when customers are awaiting service. Under this scenario, queues can build up only when there is a stock-out situation. Berman, O. and Kim, E., (1998, 1999)[8, 9] developed two Markovian models for the zero leadtime and positive leadtime cases. They determined optimal ordering policies for the discounted cost and the average cost criteria. For a given policy, Berman, O. and Sapna, K.P., (1998)[10] analyzed the problem in a non-Markovian environment under the assumption of zero leadtime. The objective was to determine the optimal stocking level in terms of the various system parameters. Berman, O. and Sapna, K. P., (1998)[11] addressed the problem of finding the optimal stocking and reorder levels that minimize the long-run expected cost rate under a prespecfied cost structure for a completely Markovian system with positive leadtime.

Berman, O., et al. (1993)[7], Berman, O. and Kim, E., (1998, 1999)[8, 9] & Berman, O. and Sapna, K. P., (1998)[11] deal with the problem of determining ordering policies or determining the optimal reorder and maximum inventory levels. However, there are practical situations where it is possible to control the service rates in order to cope up with the random fluctuations in demand or the formation of unmanageably long queues at the service facility. The dynamic control of service rates gives the system manager great flexibility in coping with the uncertainty in future demands.

An implicit assumption in most lost sales inventory models is that inventory is depleted at a rate equal to the demand rate. Though this assumption is realistic for production / manufacturing industries, it becomes unrealistic for the service facilities where inventory is necessary to perform the service. Inventory is depleted according to the demand rate when there are no customers waiting and according to the service rate when there are customers queued up for service. Examples where inventory is used in the provision of service include installing bumpers at car service stations, hospitals where units of blood are necessary for surgery and serving apple pies at restaurants.

Berman,O., Kaplan, E.H. and Shimshak, D.G., (1993)[7] considered an inventory control problem at a service facility which uses one item of inventory for each service provided. They assumed that both the demand and service rates are deterministic and constant and as such queues can form only during stock outs. They determined the optimal order quantity Q that minimizes the total cost rate.

Berman, O. and Kim, E., (1999)[9] analyzed the problem in a stochastic environment where customers arrive at service facilities according to a Poisson process and service times are exponentially distributed with mean inter-arrival time greater than the mean service time. Just as it is assumed that each service requires exactly one item from inventory. The first paper deals with the case of zero leadtime and the second one deal; with the case of positive leadtime. The main result of the papers is that under both the discounted cost case and the average cost case, the optimal policy of both the finite and infinite time horizon problems is a threshold ordering policy. The optimal policies are derived given that the order quantity is known.

Queuing systems with retrials, in which customers repeat attempts to obtain service, was originally a topic of telecommunications research. More recently, these systems have served as models for particular computer networks, which may explain the current level of activity on the subject. As an example, the "customers" of this queue could be a network of computers attempting to access the same database, which may only be used by one customer at a time.

In last two decades, many researchers in the field of retrial queuing system, contributed many results. For example, Elcan, A., (1999)[20], Arivudainambi, D., et Al. (2009)[1], Dragieva, V.I., (2013)[17], Dudin, A. N., et al.(2015a)[18] and Artalejo, J.R., et al. (2000)[6] discussed a single server retrial queue with returning customers examined by balking or Bernoulli vacations and derived analytic solutions using Matrix or Generating function or Truncation, methods using level dependent quasi-birth-and-death process (LDQBD).

Paul, M., et al. (2007)[27] and Krishnamoorthy, A., et al. (2005, 2007)[24, 25] analyzed a continuous review inventory system at a service facility with retrial of customers. In all these systems, arrival of customers form a Poisson process and service and retrial times are independent and exponentially distributed. They investigate the systems to compute performance measures and construct suitable cost functions for the optimization purpose.

For detailed survey one can see Yang, T. and Templeton, J. G. C., (1987)[30] and Falin, G.I., (1990)[21]. Choi, B. D. and Park, K. K., (1990)[13] investigated an $M/G/1$ retrial queue with two types of customers in which the service time distribution for both types of customers are the same. Khailal, Z., et al. (1992)[23] investigated the above model at Markovian level in detail. Falin, G. I., et al.(1993)[22] investigated a similar model, in which they assumed different service time distributions for both types of customers. In 1995, Choi, B. D., et al. [14] studied an $M/G/1$ retrial queueing with two types of customers and finite capacity. Choi, B. D. and Chang, Y., (1999)[12] investigated, Single server retrial queue, with two type of recurrent calls in the retrial group, in which they obtained generating function of queue lengths by using supplementary variable technique.

In 2000, Artalejo and Lopez- Herrero are concerned with the $M/G/1$ retrial queue with balking. The ergodicity condition is first investigated making use of classical mean and the limiting distribution of the number of customers in the system is determined with the help of a recursive approach based on the theory of regenerative processes. Many closed form expression are obtained when we reduce to the $M/M/1$ queue for some representative balking policies.

Artalejo, Rajagobalan and Sivasamy, (2000), are deals with the stochastic modeling of a wide class of finite retrial queueing systems in a Markovian environment. Using Matrix method they obtained the stationary distribution and first passage times.

Research Methodology

Most of the models in MDP problems are taken up for indepth study using the tools:

(i) LPP method

(ii) Policy-iteration method and

(iii) Value-iteration method

(i) Linear Programming Approach

In this thesis, MDP problems are taken up for indepth study using LPP method. Consider the objective function $f(x)$ as maximization of profit or minimization of cost, subject to the constraints involving all decision variables and quantity of resources will give the LPP formulation. Solving the problem by the standard Simplex method, We get optimal solution for the MDP problem.

(ii) Policy-Iteration

The policy-iteration algorithm converges after finite number of iterations to an average cost optimal policy. The policy-iteration algorithm is empirically found to be a remarkably robust algorithm that converges very fast in specific problems.

(iii) Value-Iteration

The value-iteration algorithm computes recursively a sequence of value functions approximating the minimal average cost per time unit. The value functions provide lower and upper bounds on the minimal average cost and under a certain aperiodicity condition these bounds converge to the minimal average cost.

Organization of the Thesis

The organization of the thesis is as follows:

Chapter 1, introduces the subject matter for research and the motivation for selection of problem. Review of literature on the topic and the related references and research objectives are also given in two sections.

Chapter 2 gives briefly the Basic concepts on Markov Decision Process, inventory control system, queueing system and Retrial queueing systems.

In chapter 3, control of service rate in a service facility system with Retrial Demands. Linear Programming method is used to control the Optimum service rates in service facility system.

In chapter 4, deals a service control in a service facility system with inventory maintenance and retrial demands using semi-MDP. The Optimum control policy to be employed is found using LPP method so that the Long-run expected cost rate is minimized.

In chapter 5, we discus service control problem in service facility with two types of customer-semi MDP. LPP method has been used to get the optimum ordering policy for perishable items in retrial service facility with instantaneous replenishment.

In chapter 6, a finite source Retrial queue with inventory management -semi MDP.Linear Programming method and Policy iteration methods are used to find the Optimum inventory policy in service facility system.

Conclusion & Scope for Future Work

Conclusion

This thesis deals with Analysis of inventory ordering control in a service facility system with retrial as a MDP is fairly recent system study. Most of the previous work determined optimal ordering policies or system performance measures. We newly introduce MDP formulation in retrial queueing system for maintaining inventory. In this research work we approach the problem to control the ordering policy (rule) via the long - run average cost criterion. We determine the optimum ordering policy to be employed to minimize the long-run expected cost rate. Thus the optimal ordering control in a continuous time semi Markov process based service facility is established.Various system performance measures are derived and the long-run expected cost rate is calculated. By assuming a suitable cost structure on the inventory system, we have presented extensive numerical illustrations to show the effect of change of values on the total expected cost rate.

Scope for Future Work

It is proposed to formulate models with more realistic assumptions which may lead us to consider advanced stochastic processes for the various behaviors of the inventory system. it is also proposed to carry out simulation studies on the models studied in the thesis when analytical solutions is lacking. In future, to get more insight into the performance of the supply chain systems, the stochastic modeling in Supply Chain has to go a long way.

List of Publications

1. *Perishable Inventory Control in Supply Chain: A Semi- MDP?*, (with Santhi, P.K. and Elango, C.), International Journal of Computational and Applied Mathematics, ISSN 1819-4966, volume 12, Number 1, pp 518-536, 2017.

2. *Optimal Control of Service Rate in a Service Facility System Maintenance and Retrial Demands?*, with Elango, C.), Intern. J. Fuzzy Mathematical Archive, 227-234 ISSN: 2320 ?3242 (P), 2320 ?3250 (online), Vol. 15, No. 2, (2018).

List of papers communicated

1. *Finite Source Retrial Queue with Inventory Management: Semi MDP*, (with Selvakumar, C. and Elango, C.), Journal of Computer and Mathematical Science (May 2019).

List of Papers Presented at Conference

1. *MDP in Supply Chain Inventory Control System*, **ICMMCMSE-2017**, organized by Alagappa University, Feb 22, 2017.

2. *Admission Control Problem in a Service Facility with inventory management*, **Recent Advancements in Pure and Applied Mathematics** oraganized by Nadar Saraswathi College of Arts and Science, Theni, July 26 - 27, 2017.

3. *Optimal Control of Service rate in a Service Facility System Maintenance and Retrial Demands*, **ICRTSMA - 2018** oraganized by Department of Statistics, Manonmaniam Sundaranar University, Tirunelveli, January 08 - 09, 2018

4. *Optimal Control of Service rate in a Service Facility System with Inventory Maintenance ? Semi MDP*, **International Seminar on Algebra and Applied Mathematics** oraganized by HKRH College, Uthamapalayam, Theni District, January 11, 2018.

5. *Service Facility with Inventory Maintenance and Retrial Demands - MDP* , **International Conference on Modern Trends in Mathematics** oraganized by HKRH College, Uthamapalayam, Theni District, September 10, 2018.

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