SYNOPSIS

MARKOV DECISION PROCESS IN SUPPLY CHAIN MANAGEMENT

This thesis deals with some stochastic inventory control problems in Supply Chain Management System (SCMS). The main objectives of the study are to find steady state probabilities of the inventory levels at various echelons of the system and the optimum values of the decision variables that minimize the total excepted cost in maintaining SCMS. Most of the results are illustrated with numerical examples and sensitivity analysis is done for various cost parameters such as holding cost, setup cost and backordering cost.

This introductory chapter contains a brief account of preliminary concepts in Inventory System and its developments, importance, classification, different types of decision making, inventory management, multi-commodity and Multi-echelon inventory system. The supply chain concept and its features in business are depicted in the literature relevant to the research work and an outline of the work done by the researcher has been given in the subsequent sections.

One of the seminal papers in the field of continuous review MEI is a basic one written by Sherbrook in 1968 [73] . He assumed that (S-1,S) polices in a Deport of – Base system force reparable item used in American Air Force (AAF). He approximated the average unit years of inventory and stock art in the Air bases. This result has been used by many subsequent researchers, since it gives a nice approximations for the lead time of the bases. The lead time of bases has computational complexity in any MEI systems.

Moinzadeh, K., and Lee, H.L., (1986) [55] considered the issue of determining the optimal batch size for ordering and stock levels of the stocking locations by using a proper approximations. Moinzedah, K., (1987) [55] generalized previous models ME repairable inventory system to cover the cases of batch ordering and batch shipment. Deuermeyer, B., and Schwarz, L.B., (1981) [19] proposed simple approximations for a complex multi – echelon system (one – warehouse and multiple retailers) assuming backordering of all unsatisfied demands in all installations with batch ordering policy. Svoronos, A. and Zipkin, P., (1988) [90] proposed several refinements as the latter paper by second moment information (SD as well as mean) in their approximation. In 1990s Axsater, S., [8] provided a simple recursive procedure for determining the holding and stock out costs of a system consisting of one central warehouse and multiple retailers with (S-1,S) policy. Demands occurred during stock outs period are back ordered at all installations with constant lead times. Axsater, S., (1990) [8] proposed exact and approximate methods for evaluating previous systems for the case of general batch size in all installations but with identical retailers.

For the case of non – identical retailers, Axsater, S., (1993) [9] proposed methods for exact evaluations of two non – identical retailers and approximate evaluation for more than two non – identical retailers.

The common assumptions of the above papers are

(1) Demands during the stock out periods are backordered.

(2) Some of the conditions demands may be lost (partial back order).

Anderson (1991) [1] and Melchiors (2001) [54] have proposed an approximate method for the case of lost sales when the inventory control policy is (S-1, S) in all installations (one – warehouse and multiple retailers) and the unsatisfied demands are last in the retailer's node.

They also introduced cost evaluations of such a system in the case of batch ordering policy as a future field of research.

Seifbarghy, M., and Jokar, M. R.A., (2005) [83] proposed a two echelon inventory system (one – warehouse and multiple retailers) with batch size determined by deterministic model with known replenishment costs at retailer and warehouse. They found optimal reorder levels, by minimizing the total holding costs of warehouse and retailers and stock out costs at the retailers.

This thesis is divided into seven chapters including this introductory chapter. (**Chapter-1)** This chapter presents some preliminary concepts in Inventory System and its developments, importance and different type of decision-making. The details of software used in the computational part of the thesis are also given.

In **Chapter-2,** the tools and techniques used to solve Markov Decision Problems are listed. Major and versatile tools used are Markov Chains (continuous and discrete time) and relevant results. (Definition, Examples and Theorems). Markov Decision Process and its solution procedures like (1) LPP (2) Policy iteration and (3) Value iteration are described in detail.

Chapter-3, deals with continuous time Markov decision processes with finite state and finite action set. The optimization criteria is uses expected average cost rate. LPP is used to solve the decision problem and optimal policy is obtained to run the supply chain management system.

In **Chapter-4**, semi Markov decision problem on SCM is studied in that which has distribution centre and RV with service facility. This chapter differs from previous one by maintaining perishable inventory in both DC and RV nodes. Inter arrival time of demands as RV is exponentially distributed with parameter λ. The service time is exponentially distributer with parameter δ. Maximum capacity of waiting space is η. Each item in RV has the perishable rate Q with exponentially distribution.

Chapter-5, deals with as a continuous time Markov decision on process with state space and finite action space. The service rate at retailer node is controlled by having decision according to the long run expected cost rate criteria. Each item in inventory has the perishing rate Q with exponentially distributed life time. Impatient customers range from the system with rate $\alpha > 0$. (Exponentially distributed inter range time).

In **Chapter-6,** we consider a different model in which inventory is controlled at retailer node but no service facility (items are issued as and when demand occurred). That is there is no queue in the retailer node. Demands that are occurring during the stock out periods are backlogged upto as specified queuing 'b' are retailer node such that the ordering quantity $Q > b + s$**.**

Chapter-7, discusses we consider a different kind of supply chain which maintains perishable inventory at the retailer node. The demand process is assumed to be Poisson with mean rate λ. Inventory at distribution centre is maintained in terms of

maximum S items is stored, where Q = S – s items are ordered at reorder point s.. The ordering policy at retailer node is (s, S) type and ordering quantity Q = S – s > b + s, where b is a batch order limit. Back logging is assumed up to a finite level say ‘b’ (>0)

The thesis ending with references, list of research papers presented / publication research papers.

ANNEXURE – I

LIST OF PUBLICATIONS

1. ***P.K. Santhi, Gowsalya, C.Elango,*** *Perishable Inventory Control in Supply Chain : A Semi-MDP Model in the International Journal of Computational and Applied Mathematics, Volume 12,Number 1, ISSN No. :1819-4966, (2017)*
2. ***P.K. Santhi, C.Elango,*** *MDP in supply chain: Inventory system with service facility at retail node with impatient customer" in Journal of Management Science and Humanities, Vol.4(2), 132-144 ,December 2017,ISSN No:2395 -0625,(2017)*
3. ***P.K. Santhi, C.Elango,*** *Optimal Inventory Control with Partial Backlogging in Supply Chain: MDP Approach " in the International Journal of Research in Information Technology, Vol.2, Issue2, Feb 2018, Pg 30-31, ISSN(online):2001-5569*
4. ***P.K. Santhi, C.Elango****, Optimal Inventory Control For Perishable Inventory with partial Backlogging in a Supply Chain: Markov Decision Process " in the International Journal of Fuzzy Mathematical Archive , Vol.15, No.2,2018, 30 th April 2018, PP No : 167-176,ISSN: 2320-3242(P),2320-3250(online).*
5. ***P.K.Santhi, C.Elango ,****MDP in Supply Chain: Optimal Inventory Control System " in the International Journal of Engineering Science Invention (IJESI), ISSN(online):2319-6734, ISSN (print) : 2319-6726,Vol.7, Issue 8,Version II, 22 nd August 2018, PP No : 55-62,Impact Factor:5.962, UGC(Approved).*

LIST OF PRESENTATIONS

1. *Paper Presented, entitled "MDP in Supply Chain Inventory Control System" in the International Conference on Mathematical Modeling and Computational Methods in Science and Engineering (ICMMCMSE- 2017) jointly organized by Ramanujan Centre for Higher Mathematics and Department of Mathematics ,Alagappa University, Karaikudi, (20th -22nd February, 2017).*

2. *Paper Presented, entitled" Perishable Inventory Control System in Supply Chain: A semi-MDP Model" in the UGC Sponsored National conference on Mathematical Modelling(NCOMM-2017) organized by PG & Research Department of Mathematics, Cardamom Planter's Association College, Bodinayakanur, Theni District,TamilNadu(30th -31st March,2017)*

3. *Paper Presented, entitled "MDP in supply chain Inventory system with service facility at retail node with impatient customers" in the two days National Conference on Recent Advancements in Pure and Applied Mathematics organised by Department of Mathematics ,Nadar Saraswathi College of Arts and Science, Vadaputhupatti Theni D.t (26th - 27th July ,2017*)

4. *Paper Presented, entitled "Inventory Control in a service Facility System is Supply Chain:Semi Markov Decision Process" in the National Seminar on Recent Trentsin Applied Mathematics organised by the Research Centre & PG Department of Mathematics ,Jayaraj Annapackiam College for Women(Autonomous), Periyakulam, Theni D.t (13th December ,2017*)

5. *Paper Presented, entitled "Optimal Inventory Control with Partial Backlogging in a Supply Chain: Markov Decision Process" in the International Conference on Recent Trends in Stochastic Modeling and its*

Applications (ICRTSMA 2018) organized by Department of Statistics ,Manonmaniam Sundaranar University, Tirunelveli, (8^{th} -9^{th} , January, 2018).

6. *Paper Presented, entitled "Optimal Control for Perishable inventory with PartialBacklogging in a Supply Chain:MDP approach" in the one day International Seminar on Algebra and Applied Mathematics organised by Department of Mathematics ,Hajee Karutha Rowther Howdia College, Uthamapalayam,Theni D.t(11^{th} January,2018*)
7. *Paper Presented, entitled "Optimal Control of Supply Chain with Service Facility having Impatient Customers" in the TNSCST sponsored International Conference on Emerging Trends in Mathematical Sciences and Technology organised by the Research Centre & PG Department of Mathematics ,Jayaraj Annapackiam College for Women(Autonomous), Periyakulam, Theni D.t (20^{th} -21^{st} December ,2018*)

REFERENCES

[1] Anderson, W. J., (1991), Continuous-time Markov Chains: An applications-Oriented Approach, Springer-Verlag, Berlin.

[2] Archibald, B. C. and Silver, E. A., (1978), (s, S) policies under continuous review and discrete compound Poisson demand, Management Sciences, 24, 899 - 909.

[3] Archibald, B. C., (1981), Continuous review (s, S) policies with lost sales, Management Sciences, 27, 1171-1177.

[4] Arivarignan, G., (1994), Perishable inventory systems with renewal demands and (−r, S) ordering policy, Published in the Proceedings of the Third Ramanujan Symposium on Stochastic Processes and Applications, held in the Ramanujan Institute for Advanced Studies in Mathematics, University of Madras during January 17-19, 1994, 211 - 220.

[5] Arivarignan, G., Elango, C. and Arumugam, N., (2002), A continuous review perishable inventory control system at service facilities, in: Artalejo, J. R., Krishnamoorthy, A., (Eds.)., Advances in Stochastic Modelling, Notable Publications, Inc., New Jersey, USA, 19 - 40.

[6] Arrow, K., Karlin, S. and Scarf, H. (1958), Studies in Applied Probability and Management Science, Standford University Press, Standford.

[7] Arrow, K., Harris, T. and Marschak, 1. (1951), Optimal Inventory Policy, Econometrica,19,3, 250-272.

[8] Axsäter, S., (1990). Simple solution procedures for a class of two-echelon inventory problems. Operations research, 38(1), 64-69.

[9] Axsäter, S., (1993). Exact and approximate evaluation of batch-ordering policies for two level inventory systems. Oper. Res. 41 777-785.

[10] Axsäter, S., (2006). Inventory Control, 2nd edition. International Series in Operations Research & Management Science. New York: Springer Science.

[11] Beckmann, M. J. and Srinivasan, S. K., (1987), An (s, S) inventory system with Poisson demands and exponential lead time, OR Spectrum, 9(4), 213- 217.

[12] Benita, M., Beamon, Supply Chain Design and Analysis: Models and Methods, University of Washington, Industrial Engineering, International Journal of Production Economics (1998), Vol. 55, No. 3, pp. 281-294.

[13] Berman, O., and Kim, E., (1999), Stochastic inventory policies for inventory management at service facilities, Stoch. Model., 15, 695-718.

[14] Berman, O., and Sapna, K. P. (1998), Inventory management at service facilities for systems with arbitrarily distributed service times, Working paper, Faculty of Management, University of Toronto.

[15] Berman, O., and Sapna, K. P., (2001), Optimal control of service for facilities holding inventory, Computers and Operations Research, 28, 429-441.

[16] Cinlar, E., (1975), Introduction to stochastic processes, Prentice-Hall, Englewood Cliff, New Jersey.

[17] Cinlar, E., (1975), Markov renewal theory - a survey, Management Sciences, 21, 727-752.

[18] Systems: Theory and Practice, L. Schwarz, Ed., Elsevier; North-Holland, Amsterdam, The Netherlands.

[19] Dvoretzky, A., Keifer, j. and Wolfowitz, J. (1952), The inventory problem I, I1, Econometrika, 20, 187-222.

[20] Elango, C., and Arivarignan, G., (2001), A lost sales inventory system with multiple reorder levels (In Russian), ElectronnoeModelirovanie, 23, 74-81.

[21] Elango, C., and Arivarignan, G., (2003), A continuous review perishable inventory systems with Poisson demand and partial backlogging, in:

Statistical Methods and Practice : Recent Advances, Balakrishnan, N. et al., (Eds.), Narosa Publishing House, New Delhi.

[22] Elango, C., (2001), A continuous review perishable inventory system at service facilities, unpublished Ph. D., Thesis, Madurai Kamaraj University, Madurai.

[23] Erharat, R., and Wagner, H. M., (1982), Inventory models and practice in: Advanced Techniques in the Practice of Operations Research, Greenberg et al., (Eds.), North Holland, New York.

[24] Erlang, A.K., The theory of probabilities and telephone conversations, Tidsskrift Matematika, B.20, (1909), pp.33-39.

[25] Federgruen, A. and Zipkin, P., (1984), An efficient algorithm for computing optimal policies, Operations Research, 32, 1268 - 1285.

[26] Federgruen. A., (1993). Centralized planning models for multi echelon inventory system under uncertainty, S.C.Gravesetal.eds. Handbooks in ORMS vol 4, North-Holland, Amsterdam, The Netherlands, 133-173.

[27] Feldman, R. M., (1975), A continuous review (s, S) inventory system in a random environment, Journal of Applied Probability, 15, 654 - 659.

[28] Graves, S. C.,(1982), The application of queuing theory to continuous perishable inventory systems, Management Science, 28,400-406.

[29] Graves, S. C., (1985). A multi-echelon inventory model for a repairable item with one-for-one replenishment. Management Science, 31(10), 1247-1256.

[30] Gross, D., and Harris C. M., (1971), On one for one ordering inventory policies with state dependent lead times, Operations Research, 19, 735-760.

[31] Harris, F., (1915), Operations and costs, Factory management series, A.W. Shah Co., Chicago, 48 - 52.

[32] He, Q.-M., Jewkes, E. M. & Buzacott, J., (1998), An efficient algorithm for computing the optimal replenishment policy for an inventoryproduction system, In A. Alfa & S. Chakravarthy (Eds.),

Advances in matrix analytic methods for stochastic models (pp. 381-402), New Jersey, NJ: Notable Publications.

[33] Kalpakam, S., and Sapna K. P. (1993), A modified (s, S) inventory system with Lost sales and emergency orders, Opsearch, 30,4, 321-336.

[34] Kalpakam, S., and Sapna K. P. (1993), A control policy of an inventory system with compound Poisson demand, Stochastic Analysis and Application, 11, 4, 459-482.

[35] Kalpakam, S., and Sapna K. P. (1994), Continuous review (s, S) inventory system with random life-times and positive lead times, Operations Research Letters, 16, 115-119.

[36] Kalpakam, S., and Sapna K. P. (1997), A lost sales inventory systems with supply uncertainty, Computers and Mathematics with Applications, 33, 3,81-93.

[37] an exhibit inventory system, Stochastic Analysis and Applications, 3, 4, 447-466.

[38] Kalpakam, S., and Arivarignan, G., (1985b), A continuous review inventory system with arbitrary inter arrival times between demands and an exhibit item subject to random failure, Opsearch, 22, 153-168.

[39] Kalpakam, S., and Arivarignan, G., (1989), On exhibit inventory system with Erlangian lifetimes under renewal demand, Ann. Inst. Stat. Math., 41, 3, 601-616.

[40] Karlin, S., and Taylor, H. M., (1975), A first course in stochastic Processes, Second edition, Academic press, New York.

[41] Karlin, S., and Taylor, H. M., (1998), An Introduction to Stochastic Modeling, Third edition, Academic press, New York.

[42] Kendall, D. G., (1953), Stochastic Processes Occuring in the Theory of Queues and Their Analysis by the Method of Imbedded Markov Chains, Ann. Math. Statist., 24, pp. 338-354.

[43] Manoharan, M., and Krishnamoorthy, A., (1989), Markov renewal theoretic analysis of a perishable inventory system, Journal of Management Sciences, 10, 2, 47-57.

[44] Medhi, J., (2009) Stochastic processes, Third edition, New Age International Publishers, New Delhi.

[45] Melchiors, P., Dekker, R., & Kleijn, M., (2000), Inventory rationing in an (s, Q) inventory model with lost sales and two demand classes, J. Oper. Res. Soc., 51 (1), 111 - 122

[46] Moinzadeh, K., & Lee, H. L. (1986). Batch size and stocking levels in multi-echelon repairable systems. Management Science, 32(12), 1567-1581.

[47] Naddor, E., (1966), Inventory System, John Wiley and Sons, NewYork.

[48] Nahmias, S., (1982), Perishable inventory theory; a review, Opsearch, 30, 680-708.

[49] Nahmias, S., and Wang, S. S., (1979), A heuristic lot size reorder point model for decaying inventories, Management Science, 25, 90-97.

[50] Nahmias, S., (1974), On the equivalence of three approximate continuous review models, Naval Research Logistics Quarterly, 23, 31 - 36.

[51] Nahmias, S., (1975), Optimal ordering policies for perishable inventory - II, Operations Research, 23, 735 - 749.

[52] Nahmias, S., (1980), Queuing modes for controlling perishable inventories, Proceedings of the First International Conference on Inventories, Hungarian Academy of Sciences, 449 - 457.

[53] Pal, M. and Ghosh, S. K., (2004), An inventory model for deteriorating items allowing shortages, IAPQR Transactions, 29(2), 121 - 128.

[54] Pal, M., (1990), An inventory model for deteriorating items when demand is random, Calcutta Statistical Association Bulletin, 39, 201 - 207.

[55] Perumal, V., Anbazhagan, N. and Arivarignan, G., (2003), Transient and Steady State Analysis of Stochastic Inventory System with a Set of Multiple Reorder Levels, ActaCienciaIndica, XXVIII, 397 - 402.

[56] Pierre Masse (1946), Les reserver et al regulation de i avenierdans la vie economique, 2-Vols, Herrnan and Cie, Parris.

[57] Porteus, E. L, (1971), On the optimality of generalized (s, S) policies, Management Sciences, 17, 411 - 426.

[58] Porteus, E. L, (1988), Stochastic inventory theory, in: Heyman, D. and Sobel, M. (Eds.), Handbook of Operations Research and Management Science, 2, North Holland, 605 - 652.

[59] Prasad S. (1994), Classification of inventory models and systems, International Journal of Production Economics, 34, 2,209-222.

[60] Qi. Ming He and E. M. Jewkes. 2000. Performance measures of a make to order inventory-production system. IIE Transactions, 32, 409-419.

[61] Raafat, F., (1991), A survey of literature on continuously deteriorating inventory models, Journal of Operational Research Society, 42, 27-37.

[62] Ravichandran, N., (1995), Stochastic analysis of a continuous review perishable inventory system with positive lead time and Poisson demand, European Journal of Operations Research, 84, 444 - 457.

[63] Richards F. R. (1975), Comments on the distribution of inventory positions in a Continuous review (s, S) inventory systems, Operations Research, 23, 366-371.

[64] Richards, F. R., (1978), A continuous review (s, S) system in a random environment, Journal of Applied Probability, 15, 654 - 659.

[65] Sahin I. (1979), On the stationary analysis of continuous review (s, S) inventory system with constant lead times, Operations Research, 27, 719-729.

[66] Sahin I. (1983), on the continuous review (s, S) inventory model under compound renewal demand random lead times, Journal of Applied Probability, 20,213-219.

[67] Sahin, I. and Sinha, D. (1987), On asymptotic approximation for (s, S) policies, Stochastic Analysis and Applications, 5, 189 - 212.

[68] Sahin, I., (1990), Regenerative inventory systems operating characteristics and optimization, Springer Verlag, New York.

[69] Sapna K.P. (1996), A continuous review (s, S) inventory system with priority customers and arbitrarily distributed lead times, Stochastic Models (edtors; Krishnamoorth et al., 213-224.

[70] Schmidt, C. P. and Nahmias, S., (1985), (S,s) policies for perishable inventory, Management Sciences, 31, 719 - 728.

[71] Seifbarghy. M, and Jokar. M.R.A, Cost evaluation of a two-echelon inventory system with lost sales and approximately normal demand. International Journal of Production Economics, 102:244–254, 2006.

[72] Sharafali, M., (1984), On a continuous review production inventory problem, Operations Research Letters, 3, 199 - 204.

[73] Sherbrooke, C., 1968. METRIC: A multi-echelon technique for recoverable item control. Operations Research. 16(1), 122 - 141.

[74] Sivazlian B. D. (1974), A continuous review (s, S) inventory system with arbitrary inter arrival distribution between unit demands, Operations Research, 22, 65-71.

[75] Srinivasan S. K. (1979), General analysis of (s, S) inventory systems with random lead times and unit demands, Journal of Math. Phy. Sci., 13, 2, 107-129.

[76] Srinivasan S. K. (1989), Analysis of (s, S) inventory systems with decaying items, Engineering costs and production economics, 15,433-439.

[77] Srinivasan, S. K., (1988), Analysis of (s, S) inventory systems with general lead time demand distributions and adjustable reorder size, Optimization, 19, 557 - 576.

[78] Svoronos, Antony and Paul Zipkin, (1991). Evaluation of One-for-One Replenishment Policies for Multiechelon Inventory Systems, Management Science, 37(1): 68-83.

[79] Thangaraj V. and Ramanarayanan R. (1983), Anopcrating policy in inventory systems with random lead times and unit demands, Math. Operations forsch U. Statist. Ser. Optim, 14111-124.

[80] Tijms H. (1972), Analysis of (8, S) inventory models, Mathematical centre Tracts 10, Amsterdam.

[81] Veinott A. F. Jr. (1966), the status of Mathematical Inventory Theory, Management Science, 12,745-777.

[82] Veinott, A. F. and Wagner, H. M., (1965), Computing optimal (s, S) inventory policies, Management Sciences, 11, 525 - 552.

[83] Wagner, H. M., (1962), Statistical management of inventory systems, John Wiley and Sons, New York.

[84] Whitin T. M. (1953), Theory of Inventory Management, Princeton University Press, Princeton.

[85] Zipkin, P. H. (1991), Model for design and control of multi item batch production systems, Operations Research, 34, 91 -104.

[86] Zipkin, P. H. (1991), Foundations of Inventory Management, Tata McGraw-Hill Company, USA.